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OPTIMAL TARIFFS WHEN PRODUCTION IS FIXED

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Abstract

The effects of tariff wars on welfare are analysed for the case of trade between two countries with fixed outputs of the traded good. Assuming mild conditions, it is shown that if there are non-zero tariffs for which welfare-maximizing equilibrium holds, then free trade is not strictly preferable when the countries' outputs are equal, and if they are not equal is strictly disadvantageous to the country with the smaller output. It is also shown that welfare-maximizing equilibria do exist if the demand function is linear.

Key Words: Commercial policy; Trade negotiations; Market Structure and Firms Strategy; Multinational Firms, Free Trade.

J.E.L. classification: F13; L1; F23.

1. Introduction[1]

It is well known that although large countries can manipulate the terms of trade to their advantage by using tariffs, the equilibrium finally reached following retaliation and successive counter-retaliations may leave all countries worse off than under free trade. For example, Baumol and Blinder (1985) assert that when every country imposes optimal tariffs on its imports, everyone is likely to lose in the long run. However, it was pointed out many years ago (Johnson 1953-54) that this outcome is not absolutely certain. With the premise that each country adopts a Cournot-type assumption regarding its rival's reaction to a change in its own tariff, Johnson reached this conclusion by considering the situation shown in Fig.1, in which x_i denotes country i's exports to country j (j=3-i), OCi is country i's offer curve under free trade (which intersects OCj at the point of free trade equilibrium), and U1 and U2, the utility isolines corresponding to the utilities achieved at free trade equilibrium, are tangential to each other at the equilibrium point. A tariff war shifts the equilibrium point from its position under free trade, but where final stable equilibrium is achieved depends on the precise nature of the offer curve and utility isolines of each country. If the equilibrium settles in region A, then country 1 gains from the tariff war; if it settles in region B, country 2 gains; if it falls in region C, both countries lose. The possibility that stable equilibrium may fall in region A or region B is of course very important for explaining the widespread existence of tariffs: a country that gains from a tariff war will not agree to free trade.

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Since Johnson's paper, many studies have investigated the question of what variables determine the achievement of stable equilibrium outside region C. Johnson himself noted the importance of the elasticities of demand and supply. Kenan and Riezman (1988) have found conditions under which a large country wins tariff wars. Chan (1991) has shown that the more dependent on trade a country is, the more likely it is to lose in a tariff bargaining game. Because of the assumptions on which their analyses were based, a common finding of these and other studies is that if the countries are mutually symmetric they will all be worse off as a consequence of the tariff war only if a country differs in some way from the other countries involved can it benefit from a tariff war. In fact, Riezman (1982) showed that free trade is not an element of the negotiation set between asymmetric countries, and is therefore never the outcome. However, in this paper we show that there are conditions under which free trade is never more advantageous to all countries than an equilibrium with non-zero tariffs, even if the countries are symmetric.

We note that the imposition of optimal tariffs is not necessarily optimal policy from a global perspective (Kaempfer 1989), and that, contrariwise, it may be justified by external factors such as the exploitation of international market power in a context of imperfect competition (Brander and Spencer 1985). For surveys of literature clarifying the significance of imperfect competition for trade policy, see for example Helpman and Krugman (1989) and Harris (1989).

2. The model

Like Brander (1981) and Brander and Krugman (1983), we assume that two countries, 1 and 2, produce outputs e_i (i=1,2) of an homogeneous good that is sold entirely in the markets of the two producing countries. In this paper, both the outputs e_i are fixed, i.e. the output of neither country depends explicitly on either its exports to or its imports from the other. Denoting the exports of country i by x_i, the quantities sold by each country in each market are therefore given by the following table.

	Market 1	Market 2
Country 1	$e_1 - x_1$	x_1
Country 2	x_2	$e_2 - x_2$
Total market quantity	$e_1 - x_1 + x_2$	$e_2 - x_2 + x_1$

The hypothesis that output is independent of imports and exports is restrictive, but would appear to be applicable when the volume of production is to a large extent determined by factors beyond the producers' control, as in agriculture and fishing. It is also, of course, valid generally in the short term. The effect of this hypothesis is to integrate the two markets by making the production destined for the home market depend on exports. For an analysis of integrated and segmented markets, see Venables (1990).

The prices of the good in each market are assumed to be given as a function of market quantity q by an inverse demand function $p(q)$ that is common to both markets. We denote by $p_i = p_i(q_i)$ the actual prices in market i, being $q_i = e_i - x_i + x_j$.

The only assumption made concerning the demand function is that

$$p'_k(q_k) + q_{ki}p''_k(q_k) < 0 \tag{2.1}$$

where q_{ki} is the output placed by country i in market k. This is a fairly standard regularity condition in noncooperative models (see, e.g., Brander and Spencer (1985) and Brander and Krugman (1983)); although it can be violated if the demand curve is very convex, violation is considered unusual. Assumption 2.1 is equivalent to the four inequalities

$$\begin{aligned} (e_1 - x_1)p''_1 + p'_1 &< 0 \quad (2.2) \\ x_2p''_1 + p'_1 &< 0 \\ (e_2 - x_2)p''_2 + p'_2 &< 0 \\ x_1p''_2 + p'_2 &< 0 \end{aligned}$$

and implies that

$$p'_i < 0 \quad (i = 1, 2). \tag{2.3}$$

From the above table, the profits π^i made by each country's producers, given the tariffs t_i imposed by each country i on imports, are given by

$$\begin{aligned} \pi^1(x_1, x_2, t_1, t_2) &= (e_1 - x_1)p_1 + x_1(p_2 - t_2) - c_1 \\ \pi^2(x_1, x_2, t_1, t_2) &= (e_2 - x_2)p_2 + x_2(p_1 - t_1) - c_2 \end{aligned} \tag{2.4}$$

where the c_i are the constant costs associated with the outputs e_i. In most of what follows it is assumed that both x_1 and x_2 are positive, i.e. that any tariffs are non-prohibitive. Under these conditions, the first-order Cournot-Nash conditions for profit maximization are[2]

$$\begin{aligned} \pi^1_{x_1}(x_1, x_2, t_1, t_2) &= -(e_1 - x_1)p_1' - p_1 + x_1 p_2' + (p_2 - t_2) = 0 \\ \pi^2_{x_2}(x_1, x_2, t_1, t_2) &= -(e_2 - x_2)p_2' - p_2 + x_2 p_1' + (p_1 - t_1) = 0 \end{aligned} \tag{2.5}$$

The second order conditions,

$$\begin{aligned} \pi^1_{x_1 x_1} &= (e_1 - x_1)p_1'' + 2p_1' + x_1 p_2'' + 2p_2' < 0 \\ \pi^2_{x_2 x_2} &= (e_2 - x_2)p_2'' + 2p_2' + x_2 p_1'' + 2p_1' < 0 \end{aligned} \tag{2.6}$$

are satisfied by virtue of 2.2 and 2.3, which also imply that the mixed derivatives,

$$\begin{aligned} \pi^1_{x_1 x_2} &= -(e_1 - x_1)p_1'' - p_1' - x_1 p_2'' - p_2' = -\pi^1_{x_1 x_1} + p_1' + p_2' \\ \pi^1_{x_2 x_1} &= -(e_2 - x_2)p_2'' - p_2' - x_2 p_1'' - p_1' = -\pi^2_{x_2 x_2} + p_1' + p_2' \end{aligned} \tag{2.7}$$

[2] From here on derivatives are denoted by subscripts except for the derivatives of the inverse demand function.

satisfy

$$\pi^1_{x_1x_2} > 0,\ \pi^2_{x_2x_1} > 0 \tag{2.8}$$

We define

$$\begin{aligned} D &= \pi^1_{x_1x_1}\pi^2_{x_2x_2} - \pi^1_{x_1x_2}\pi^2_{x_2x_1} \\ &= \pi^1_{x_1x_1}\pi^2_{x_2x_2} - (-\pi^1_{x_1x_1} + p_1' + p_2')(-\pi^2_{x_2x_2} + p_1' + p_2') \\ &= -(p_1' + p_2')(\pi^1_{x_1x_2} - \pi^2_{x_2x_2}) \\ &= (p_1' + p_2')\{3(p_1' + p_2') + q_1p_1'' + q_2p_2''\} \text{ (by 2.6, 2.7)} \\ &> 0 \text{ (by 2.1, 2.3)} \end{aligned} \tag{2.9}$$

Since the π^i are defined on a convex set $([0, e_1] \times [0, e_2])$, inequalities 2.6 and 2.9 imply the existence and global uniqueness of equilibrium [see Nikaido (1968)]. Note that if the equilibrium point lies on the boundary of $[0, e_1] \times [0, e_2]$, the first-order conditions may not be satisfied; in what follows, this situation will be pointed out explicitly whenever it arises.

3. Effects of tariff changes on profit-maximizing equilibria

Since $D \neq 0$, application of the implicit function theorem to the first-order conditions, considered as functions of (x_1, x_2, t_1, t_2), shows that the equilibrium exports x_i^e are locally functions of the tariffs t_i. Thus differentiating the identities

$\pi^i_{x_i}(x^e_1(t_1,t_2), x^e_2(t_1,t_2), t_1, t_2) = 0$ affords the equations

$$\begin{pmatrix} \pi^1_{x_1x_1} & \pi^1_{x_1x_2} & 0 & 0 \\ \pi^2_{x_2x_1} & \pi^2_{x_2x_2} & 0 & 0 \\ 0 & 0 & \pi^1_{x_1x_1} & \pi^1_{x_1x_2} \\ 0 & 0 & \pi^2_{x_2x_1} & \pi^2_{x_2x_2} \end{pmatrix} \begin{pmatrix} x_{1t_1} \\ x_{2t_1} \\ x_{1t_2} \\ x_{2t_2} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \tag{3.1}$$

(in 3.1 the superscript e marking equilibrium values has been dropped, as it will be hereafter). Solving, and taking 2.6, 2.8 and 2.9 into account,

$$\begin{aligned} x_{1t_1} &= -\pi^1_{x_1x_2}/D < 0 \\ x_{2t_1} &= \pi^1_{x_1x_1}/D < 0 \\ x_{1t_2} &= \pi^2_{x_2x_2}/D < 0 \\ x_{2t_2} &= -\pi^2_{x_2x_1}/D < 0 \end{aligned} \tag{3.2}$$

Proposition 1.

a) A marginal change in either country's tariff causes changes of opposite sign in each country's imports and exports.

b) By increasing (reducing) its tariff, a country reduces (increases) its exports less than its imports.

Proof. Part (a) is the content of inequalities 3.2 . Part (b) means that $x_{it_i} > x_{jt_i}$, i.e. that $-\pi^i_{x_ix_j} > \pi^i_{x_ix_i}$ (from 3.2), which follows from 2.7 and 2.3. ■

The effects of marginal tariff changes on prices in market 1 are given by

$$\begin{aligned} \partial p_1/\partial t_1 &= (-x_{1t_1} + x_{2t_1})p_1' \\ &= (1/D)(\pi^1_{x_1x_2} + \pi^1_{x_1x_1})p_1' \\ &= (1/D)(p_1' + p_2')p_1' \text{ (by 2.6, 2.7)} \\ &> 0 \end{aligned} \tag{3.3}$$

and

$$\begin{aligned} \partial p_1/\partial t_2 &= (-x_1t_2 + x_2t_2)p_1' \\ &= -(1/D)(\pi^2_{x_2x_2} + \pi^2_{x_2x_1})p_1' \\ &= -(1/D)(p_1' + p_2')p_1' \\ &= -\partial p_1/\partial t_1 < 0 \end{aligned} \tag{3.4}$$

Symmetric results hold for the effects on country 2's market price. This proves part (a) of the following proposition.

Proposition 2.

a) A country that marginally increases its tariff increases the price in its own market (and hence reduces the quantity of that market), and causes changes of equal magnitude but opposite sign in the price and quantity of the other country's market.

b) If both countries change their tariffs by the same amount $\triangle t$, there will be no change in the price or quantity of either market, but each country's exports will *change by* $\frac{\triangle t}{p_1'+p_2'}$.

Proof of part (b). That the market prices and volumes do not change follows immediately from part (a). Denoting by x the change in each country's equilibrium exports (the change is the same for both countries because the market demand $q_1 = e_1 - x_1 + x_2$ does not change), this quantity can be calculated by considering the first-order conditions on $\pi^1_{x_1}$ at (x_1, x_2, t_1, t_2) and at$(x_1 + \triangle x, x_2 + \triangle x, t_1 + \triangle t, t_2 + \triangle t)$:

$$\begin{aligned} -(e_1 - x_1)p_1' - p_1 + x_1 p_2' + (p_2 - t_2) &= 0 \qquad (3.5) \\ -(e_1 - x_1 - \triangle x)p_1' - p_1 + (x1 + \triangle x)p_2' + (p_2 - t_2 - \triangle t) &= 0 \end{aligned}$$

Subtracting and re-arranging, $\triangle x = \triangle t/(p_1' + p_2')$. ■

Whether country i increases its producers' profits if it raises its tariff on imports depends on whether p_i is greater or less than $p_j - t_j$, for

$$\begin{aligned} \pi^i_{t_i} &= \pi^i_{x_i} x_{it_i} + \pi^i_{x_j} x_{jt_i} + \partial\pi^i/\partial t_i \qquad (3.6) \\ &= \pi^i_{x_j} x_{jt_i} \quad \text{(by 2.4 and by 2.5)} \\ &= \left[(e_i - x_i)p_i' - x_i p_j'\right] x_{jt_i} \\ &= (p_j - t_j - p_i)x_{jt_i} \quad \text{(by 2.5)} \end{aligned}$$

and the latter expression is positive if $p_i > p_j - t_j$ and negative if $p_i < p_j - t_j$. The dependence of a country's producers' profits on changes in the tariff of the other country depends on both the sign of $(p_j - t_j - p_i)$ and the absolute level of the producers' exports:

$$\begin{aligned} \pi^i_{t_j} &= \pi^i_{x_i} x_{i\,t_j} + \pi^i_{x_j} x_{j\,t_j} + \partial\pi^i/\partial t_j \\ &= (p_j - t_j - p_i) x_{j\,t_j} - x_i \end{aligned} \tag{3.7}$$

4. Welfare-maximizing equilibria

We now discuss the effects of tariffs when countries desire to optimize the overall welfare derived from the good, rather than just their producers' profits. Specifically, we assume that the two countries' common price function p is the derivative of a common utility function $U(q)$, and we consider welfare functions

$$u^i(t_1, t_2) = U(q_i) - p_i q_i + \pi^i(x_1, x_2, t_1, t_2) + t_i x_j \tag{4.1}$$

Our welfare concept thus embraces consumers' surplus ($U(q_i) - p_i q_i$) and tariff income $t_i x_j$, as well as producers' profits. To investigate what values of t_1 and t_2 afford welfare-maximizing equilibria, we need the following lemma concerning profit-maximizing equilibria and the derivatives of the u^i when equal tariffs are imposed.

Lemma 1. *If both countries impose the same tariff t, then:*

a) their market prices and quantities at profit-maximizing equilibrium are also equal, and *b)* $u^1_{t_1}(t,t) = -2(\bar{p}')^2\bar{q}/D + x_2$

$u^1_{t_2}(t,t) = 2(\bar{p}')^2\bar{q}/D - x_1$

$u^2_{t_1}(t,t) = 2(\bar{p}')^2\bar{q}/D - x_2$

$u^2_{t_2}(t,t) = -2(\bar{p}')^2\bar{q}/D + x_1$

Proof. a) If $x_1 = e_1/2$ and $x_2 = e_2/2$, then $q_i = (e_1 + e_2)/2$ $(i = 1, 2)$ and the p_i are therefore also equal. Hence under free trade ($t_1 = t_2 = 0$), the values $x_1 = e_1/2$ and $x_2 = e_2/2$ satisfy the first-order conditions for profit equilibrium, 2.5, and by the uniqueness of equilibrium these are the only values affording equilibrium. Part (a) thus follows from part (b) of Proposition 2. In what follows we write $\bar{q}$ for $(e_1+e_2)/2$ and $\bar{p}$ and $\bar{p}'$ for the values of p and p' at $\bar{q}$.

b)From 4.1, and using 3.6,

$$
\begin{aligned}
u^1_{t_1}(t_1,t_2) &= p_1 q_{1t_1} - p'_1 q_{1t_1} q_1 - p_1 q_{1t_1} + (p_2 - t_2 - p_1)x_{2t_1} + t_1 x_{2t_1} + x_2 \\
&= -p'_1 q_1(\pi^1_{x_1x_1} + \pi^1_{x_1x_2})/D + (p_2 - t_2 - p_1 + t_1)x_{2t_1} + x_2 \\
&= -p'_1 q_1(p'_1 + p'_2)/D + (p_2 - t_2 - p_1 + t_1)x_{2t_1} + x_2
\end{aligned}
$$

Taking $t_1 = t_2 = t$, so that $q_1 = q_2 = \bar{q}$ and $p_1 = p_2 = \bar{p}$ (by part(a)), $u^1_{t_1}(t,t) = -2(\bar{p}')^2\bar{q}/D + x_2$. The other formulae of part (b) follow from analogous calculations. ■

Proposition 3. *Welfare-maximizing equilibrium is incompatible with free trade.*

Proof. We assume, without loss of generality, that $e_1 \leq e_2$. By the lemma,

$$u^1_{t_1}(0,0) - u^2_{t_2}(0,0) = x_2 - x_1 = e_2/2 - e_1/2 \geq 0 \tag{4.2}$$

Also by the lemma,

$$\begin{aligned} u^1_{t_1}(0,0) + u^2_{t_2}(0,0) &= -4(\bar{p}')^2\bar{q}/D + \bar{q} \\ &= \bar{q}(2\bar{p}' + \bar{p}''\bar{q})/(3\bar{p}' + \bar{p}''\bar{q}) \ \text{(by2.9)} \\ &> 0 \ \text{(by2.1 and 2.3)} \end{aligned} \tag{4.3}$$

Adding 4.2 and 4.3,

$$u^1_{t_1}(0,0) > 0 \tag{4.4}$$

Thus the free trade point does not satisfy the first-order conditions for welfare-maximizing equilibrium; given free trade, it is to the advantage of the country with the smaller output to impose a tariff. ■

Proposition 4. *If $e_1 < e_2$, free trade affords country 1 strictly less welfare than a welfare-maximizing equilibrium with at least one non-zero tariff, if this equilibrium exists.*

Proof. We first consider the total derivative of country 1's welfare with respect to a common tariff t imposed by both countries. By the lemma,

$$u_t^1(t,t) = u_{t_1}^1(t,t) + u_{t_2}^1(t,t) = x_2 - x_1 > 0 \tag{4.5}$$

Since 4.5 holds so long as $t \leq t^*$, where t^* is such that $x_1(t^*, t^*) = 0$,

$$u^1(0,0) < u^1(t,t) \quad (0 < t \leq t^*) \tag{4.6}$$

To prove the proposition, there are three cases to consider.

1) $0 < t_2 \leq t^*$. In this case, $u^1(0,0) < u^1(t_2,t_2)$ (by 4.6), and if (t_1,t_2) is an equilibrium point, $u^1(t_2,t_2) \leq u^1(t_1,t_2)$ (by the properties of equilibrium). Hence $u^1(0,0)$ is strictly less than $u^1(t_1,t_2)$.

2) $t_2 = 0$. Since $u_{t_1}^1(0,0) > 0$ (by 4.4), there exists $\varepsilon > 0$ such that $u^1(0,0) < u^1(\varepsilon,0)$. As before, $u^1(\varepsilon,0) \leq u^1(t_1,0)$, so $u^1(0,0)$ is strictly less than $u^1(t_1,t_2)$.

3) $t^2 > t^*$. As before, $u^1(t_1,t_2) \geq u^1(t^*,t_2)$. But for all $t > t^*$, $u^1(t^*,t) = u^1(t^*,t^*)$ (since increasing a tariff on non-existent imports can have no effect in either country; in this region of the (t_1,t_2) plane the results of Sections 2 and 3 no longer hold). Since $u^1(t^*,t^*) > u^1(0,0)$ (by 4.6), $u^1(0,0)$ is strictly less than $u^1(t_1,t_2)$. ■

Proposition 5. *If $e_1 = e_2$, neither country strictly prefers free trade to a welfare-maximizing equilibrium with non-zero tariffs, if this exists.*

Proof. The proof is the same as for Proposition 4, except that, since $e_1 = e_2$, $u^i_t(t,t) = 0$ $(i = 1, 2)$, so that

$$u^i(0,0) = u^i(t^*, t^*)$$

and only the weak dominance of (t_1, t_2) over $(0, 0)$ is proved: $u^i(0,0) \leq u^i(t_1, t_2)$. ■

5. Linear price functions

In Section 4 we obtained results relating welfare under free trade to welfare at welfare-maximizing equilibrium under the assumption that such equilibria existed. We now prove an existence theorem.

Theorem 5.1. *If the price function is linear,*

$$p(q) = a - bq,$$

where the constants a and b are both non-negative, then there exist t_1 and t_2, at least one of which is positive, such that welfare-maximizing equilibrium holds at the point (t_1, t_2).

Proof. We first note that condition 2.1 is satisfied, and proceed to display some equilibrium points.

To show that equilibrium holds at a point (t_1, t_2) in Q_1, the first quadrant of the (t_1, t_2) plane, where t_1 and t_2 are both non-negative, we need to investigate the behaviour of the welfare functions u^i throughout this region. Since the u^i depend on the profit-maximizing x_i, and since the functions $x_i(t_1, t_2)$ are only piecewise smooth (not being able to take negative values), we must first determine the areas of Q_1 in which these functions are smooth, which are at most four: one in which both x_i are non-negative, which we denote by Ω ; one in which x_1 is zero but x_2 non-negative (X); one in which x_2 is zero but x_1 non-negative (Y); and one in which both x_1 and x_2 are zero (Z). Depending on the relative values of e_1 and e_2, one or more of these regions may lie outside Q_1.

In Ω, for given t_1 and t_2, the first-order conditions for profit-maximizing equilibria are 2.5; since $q_i = e_i - x_i + x_j$, they constitute a pair of linear equations in x_1 and x_2, the solutions of which are

$$\begin{aligned} x_1 &= e_1/2 - (t_1 + 2t_2)/(6b) \\ x_2 &= e_2/2 - (t_2 + 2t_1)/(6b) \end{aligned} \tag{5.1}$$

Setting the x_i equal to zero in 5.1 defines two lines in the (t_1, t_2) plane. Depending on the position of their intersection point,

$$T_0 = (t_{01}, t_{02}) = (2be_2 - be_1, 2be_1 - be_2)$$

one or both of these lines constitute the boundary of Ω. If we assume, without loss of generality, that $e_1 \leq e_2$, then the only possibilities are that T_0 lies in the first quadrant (if $e_2 \leq 2e_1$; Fig.2) or the second (if $e_2 > 2e_1$; Fig.3).

If (t_1, t_2) lies in the region labelled X in Figs.2 and 3, then $x_1 = 0$ and the profit functions reduce to

$$\begin{aligned} \pi^1(0, x_2; t_1, t_2) &= (a - be_1 - bx_2)e_1 \\ \pi^2(0, x_2; t_1, t_2) &= (a - be_2 + bx_2)(e_2 - x_2) + (a - be_1 - bx_2 - t_1)x_2 \end{aligned} \tag{5.2}$$

The first-order condition on x_1 is satisfied identically, and the condition on x_2, $\pi^2_{x_2} = 0$, yields

$$x_2 = (2be_2 - be_1 - t_1)/(4b) = (t_{01} - t_1)/(4b) \tag{5.3a}$$

Similarly, in region Y in Fig.2, $x_2 = 0$ and x_1 is a function of t_2 alone:

$$x_1 = (2be_1 - be_2 - t_2)/(4b) = (t_{02} - t_2)/(4b) \tag{5.3b}$$

Only in region Z are the x_i both zero.

In view of 5.1, in Ω the welfare functions 4.1 are

$$\begin{aligned} u^1(t_1, t_2) &= (-6e_1bt_1 + 72ae_1b - 72c_1b - 27e_1^2b^2 - 19t_1^2 + 2t_1t_2 + 17t_2^2 \\ &\quad +9e_2^2b^2 - 18e_1b^2e_2 - 30e_1bt_2 + 30t_1e_2b + 6t_2e_2b)/72b \\ u^2(t_1, t_2) &= (6e_1bt_1 + 72ae_2b - 72c_2b + 9e_1^2b^2 + 17t_1^2 + 2t_1t_2 - 19t_2^2 \\ &\quad -27e_2^2b^2 - 18e_1b^2e_2 + 30e_1bt_2 - 30t_1e_2b - 6t_2e_2b)/72b \end{aligned} \tag{5.4}$$

The first-order conditions for welfare maximizing equilibrium, $u^i_{ti} = 0$, lead to the equations

$$t_2 = 3be_1 - 15be_2 + 19t_1 \quad (5.5a)$$

$$t_2 = (15be_1 - 3be_2 + t_1)/19 \quad (5.5b)$$

The line defined by equation 5.5a intersects the t_1-axis at $X_B = (15be_2 - 3be_1, 0)$ (which is always on the positive t_1-axis because $e_1 \leq e_2$), and the line $x_1 = 0$ at $B = (B_1, B_2) = (b/13)(10e_2 - e_1, 20e_1 - 5e_2)$ (which lies on the boundary of Ω if $e_2 \leq 2e_1$ as in Fig.4 (because $B_1 \leq t_{01}$ if $e_2 > (\frac{3}{4})e_1$) or if $2e_1 < e_2 \leq 4e_1$ (Figs.5-7)). It separates the region of Ω in which $u^1_{t_1} < 0$ from the region in which $u^1_{t_1} > 0$; and since $u^1_{t_1 t_1} = -19/(36b) < 0$ throughout Ω, the former region lies below and to the right of $X_B B$, and the latter above and to the left.

The line defined by 5.5b intersects the t_2-axis at $Y_C = (0, 15be_1 - 3be_2)$ (which is on the positive t_2-axis if $e_2 < 5e_1$, as in Figs.4-7), the line $x_1 = 0$ at $C = (C_1, C_2) = (b/7)(9e_1 + 2e_2, 6e_1 - e_2)$, and the line $x_2 = 0$ at $C' = (b/13)(20e_2 - 5e_1, 10e_1 - e_2)$. If $e_2 \leq 2e_1$, the line 5.5b can intersect the boundary of Ω either to the left of T_0 (as shown in Fig.4) or to the right (if $e_2 \leq (4/3)e_1$), but in any case it intersects the line 5.5a within Ω (and hence to the left of B), as it also does if $2e_1 < e_2 \leq (31/11)e_1$ (Fig.5). If $(31/11)e_1 < e_2 < 4e_1$, C lies to the left of B (Figs.6 and 7). In the (e_1, e_2) region in which B does not lie on the boundary of Ω ($e_2 > 4e_1$), C does lie on the boundary of Ω if $e_2 \leq 6e_1$, as in Fig.8, but not otherwise. Whenever the line 5.5b intersects the boundary of Ω, it separates the region of Ω in which $u^2_{t_2} < 0$ from the region in which $u^2_{t_2} > 0$; and since $u^2_{t_2t_2} = -19/(36b) < 0$ throughout Ω, the former region lies above it and to its left (X_0CY_C in Figs.4-7) and the latter below it and to its right.

In X, where $x_1 = 0$ everywhere and the profit functions are given by 5.2 and x_2 by 5.3a, the welfare functions 4.1 reduce to

$$
\begin{aligned}
u^1(t_1, t_2) &= ae_1 - \frac{3}{16}e_1t_1 - \frac{19}{32}be_1e_2 + \frac{1}{8}be_2^2 - \frac{7}{32b}t_1^2 + \frac{3}{8}e_2t_1 \\
u^2(t_1, t_2) &= ae_2 - be_2^2/2 + (t_{01} - t_1)(e_2 - e_1)/4 - 3(t_{01} - t_1)^2/(32b) - (t_{01} - t_1)/(4b)
\end{aligned}
$$

The first-order condition on u^2 is therefore satisfied everywhere in X. The first-order condition on u^1 is

$$u^1_{t_1} = (-1/(16b))(3be_1 - 6be_2 + 7t_1) = (-1/(16b))(7t_1 - 3t_{01}) = 0 \qquad (5.6)$$

Thus u^1 is negative to the left of the line $t_1 = (3/7)t_{01}$, and positive to the right. This line intersects the line X_0T_0 at a point $A = (t_{A_1}, t_{A_2})$ that is always to the left of B when B lies on the boundary of Ω ($e_2 \leq 4e_1$; Figs.4-7) but is to the right of C if $e_2 > 3e_1$ (Fig.7).

Similar reasoning shows that in Y $u^1_{t_1}$ is everywhere zero and $u^2_{t_2} = (-1/16b)(t_{02} - t_2) < 0$, and that in Z both $u^1_{t_1}$ and $u^2_{t_2}$ are everywhere zero.

In view of the above, when the intersection of lines 5.5a and 5.5b lies in Ω ($e_2 \leq (31/11)e_1$; Figs.4 and 5), it is a point of welfare equilibrium, since neither country can gain in welfare by deviating unilaterally from it. The same is true of any point on the segment CB when $(31/11)e_1 < e_2 \leq 3e_1$(Fig.6), and of any point on the segment AB when $3e_1 < e_2 \leq 4e_1$ (Fig.7). Finally, if $e_2 > 4e_1$, any point on the line $t_1 = t_{A_1}$ is a point of welfare equilibrium. ■

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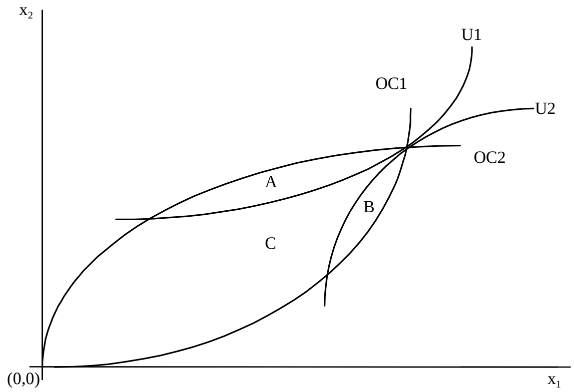

Fig. 1

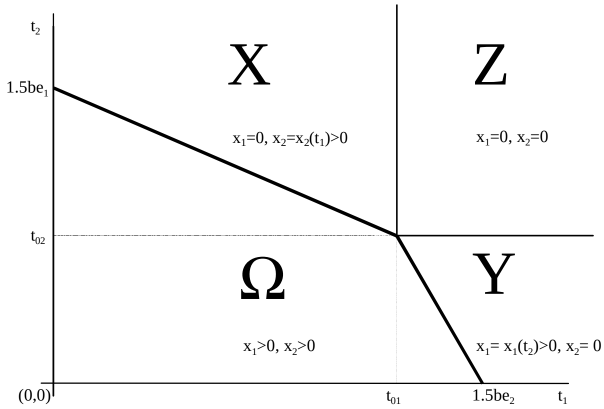

Fig. 2: $e_2 \leq 2e_1$

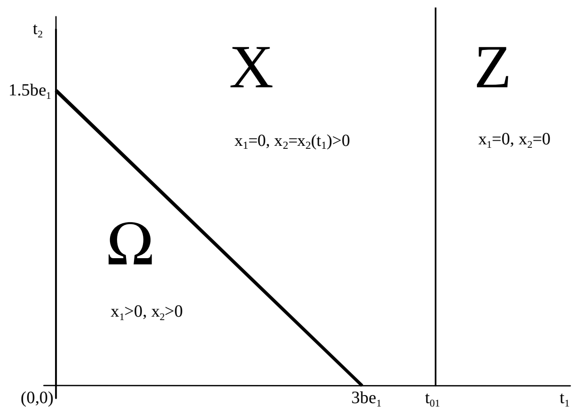

Fig. 3: $e_2 > 2e_1$

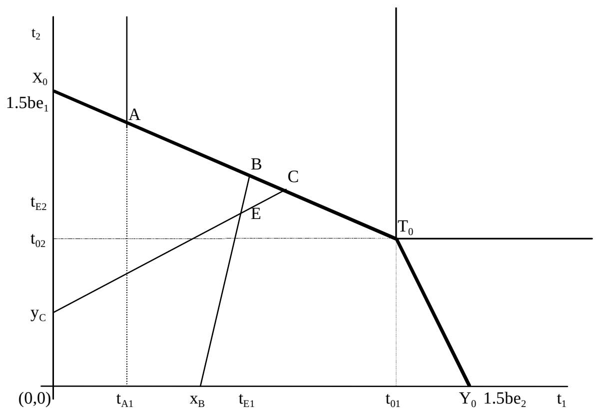

Fig. 4: $e_2 \leq 2e_1$ ($Y_C C$ is shown for $e_2 > \frac{4}{3}e_1$)

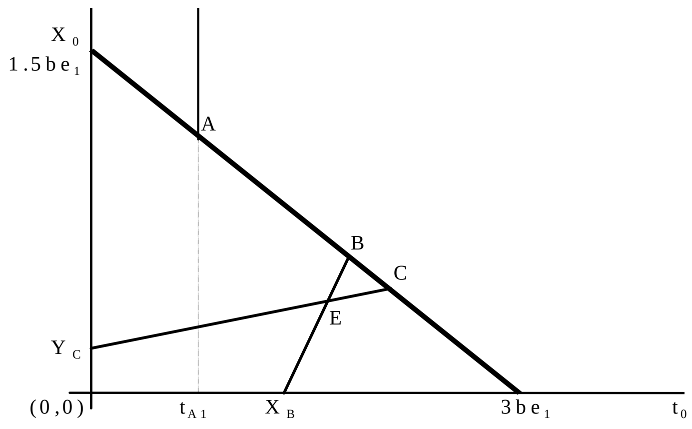

Fig. 5: $2e_1 < e_2 \leq \frac{31}{11}e_1$

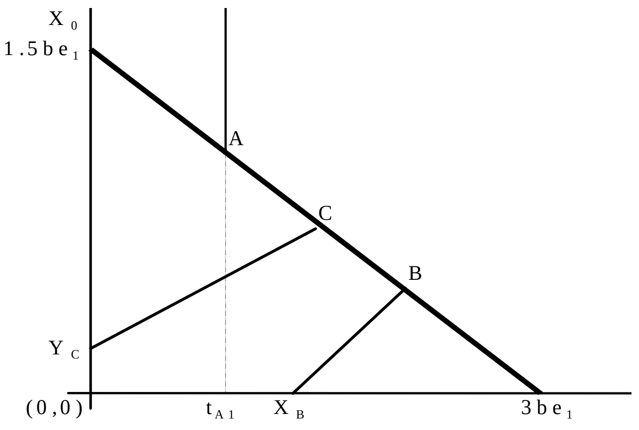

Fig 6: $\frac{31}{11}e_1 < e_2 \leq 3e_1$

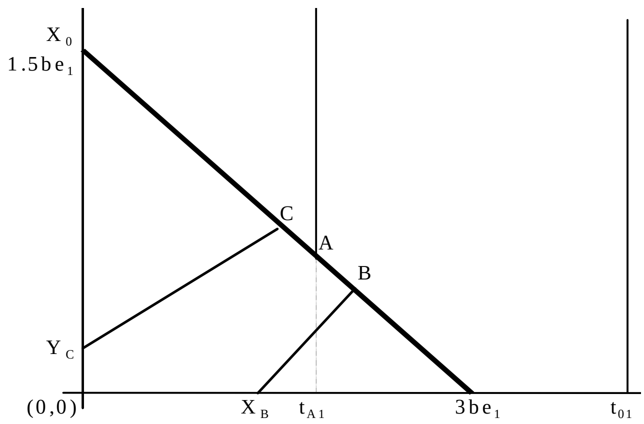

Fig. 7: $3e_1 < e_2 \leq 4e_1$

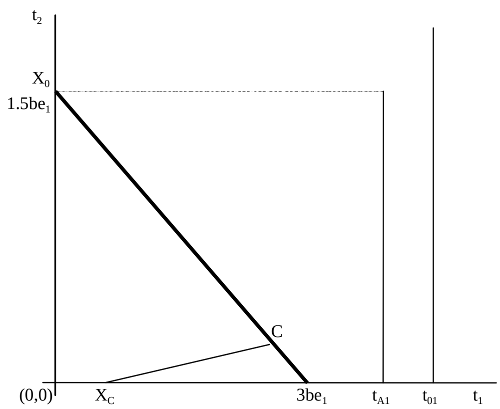

Fig. 8: $e_2 > 4e_1$ ($X_C C$ is shown for $e_2 \leq 6e_1$)

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