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**A COMPARISON BETWEEN STATISTICAL AND WALRASIAN
APPROACHES: AN APPLICATION TO A LABOUR MARKET**

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A Comparison Between Statistical and Walrasian Approaches: An Application to a Labour Market

Pedro María Rey Araújo *

ABSTRACT

The Walrasian construction of the tatonnement constitutes the inner core of the neoclassical paradigm. Despite being a satisfactory one in many cases, there are several real-world phenomena which cannot be critically appraised under this approach. We argue that the Walrasian understanding of the market implicitly limits the scope of problems that can be analysed within the paradigm. The statistical markets approach, first developed by Duncan Foley, aims at offer a distinct perspective when analysing economic interactions through the abandonment of methodological individualism. We draw a critical comparison between the two approaches in the context of a labour market. We express the same market configuration under the two approaches, thus ensuring a higher degree of comparability between the two. We check whether their predictions are similar in a number of cases, stressing those features that exclusively pertain to the statistical approach that, in our opinion, enhance the theoretical appraisal of economic activity in contemporary societies.

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1. INTRODUCTION

The Walrasian notion of the *auctioneer*, odd as it may be, has informed neoclassical economics' theoretical construction of the market ever since. Deeply informed by a Newtonian world-view, the notion of equilibrium has functioned as the very cornerstone of the theory. Strikingly, many among the most pervasive social phenomena characteristic of market economies need to be accounted for through an *ad hoc* addition of further hypothesis such as informational asymmetries, strategic behaviour or statistical discrimination, that do not belong to the very core of the theory. As Yakovenko and Barkley Rosser (2009) have pointed out, 'most of the mathematical apparatus transferred to economics from physics was that of Newtonian physics and classical thermodynamics [which] culminated in the neoclassical concept of equilibrium where the forces of supply and demand balance each other', that results in a deterministic and static conception of economic interactions.

Foley's (1994) seminal contribution shifted the focus towards statistical mechanics, thus substituting the Walrasian concept of equilibrium as a fixed point in a Euclidean space for an understanding of the equilibrium as a collection of probability distributions, i.e. a *statistical market equilibrium*. Crucially, the notion of equilibrium resulting from this framework will be dynamic rather than static, while not repressing an uncertainty endogenous to the market itself. This way, a high degree of uncertainty at the micro-level is integrated into a framework that produces stable macro-regularities through the operation of probabilistic combinations.

The statistical conception of markets remains much closer to its understanding by Classical Political Economists, who saw 'the gravitation of the market price under the natural price as a never-ending fluctuation' (Foley, 2006). Indeed, Walrasian equilibria are special cases of statistical equilibria, the former being fully contained within the latter (Toda, 2015), that is, the latter can explain everything that can be explained by the former, but not vice versa (Toda, 2015). This paper adds up to the existing literature of statistical markets in two different directions.

On the one hand, this paper explores this connection between Walrasian and statistical conceptions of equilibrium in the context of a simple labour market. In order to do that, we first build a Walrasian model of a labour market. After the properties of the Walrasian model have been explored, we employ all the information we had previously used to construct the former in order to build up a statistical representation of

the same market, so that the former functions as a primitive of the latter. This way, both models are commensurable and a direct comparison between the two becomes possible. We show that the statistical modelling can replicate all the results shown by the Walrasian construction after exogenous changes in the defining parameters, while at the same time displaying both horizontal inequality and involuntary unemployment as endogenous features of market interactions. Therefore, these results confirm the assertion that Walrasian equilibria are but special cases of statistical equilibria, towards which the latter only approaches asymptotically (Foley, 1999; Toda 2010, 2015).

On the other hand, this paper contributes to the existing literature about the examination of labour markets through the lenses of the statistical equilibrium apparatus, whose seminal contribution is to be found in Foley (1996a). We improve Foley's (1996a) account on two directions. On the one hand, we derive the statistical model from a previously constructed Walrasian one instead of building it anew, thus allowing for a direct comparison in commensurable terms in a way not allowed by Foley. On the other hand, our conceptualization of the households' offer set is more complex, as we introduce a continuous labour supply, with a linearly increasing reservation wage for each level of labour-power provided to the market. Further, we introduce an upper limit to their offer set instead of introducing a declining density of points, so that the households already incorporate some information relative to the firms' production process. Another contribution very similar to ours in its motivation can be found in De Castro Soromenho (2011). There, an explicit application of the statistical equilibrium apparatus to a labour market is accomplished as well. However, whereas it does so in the context of Kaleckian-inspired macroeconomic labour market, we prefer to remain at a strictly micro level in order to make the comparison with the Walrasian approach clearer.

Perhaps, the application that remains closer to ours is Akira Toda (2010). He starts as well by conceiving a Walrasian model that works as a primitive, in the framework of a primitive agricultural economy. However, the present paper differs in that its motivation is to ascertain what the terms would be of a straightforward comparison to the Walrasian model, whereas Akira Toda's contribution remains committed to explore the interiority of the statistical paradigm.

The present is organized as follows. Section 2 outlines two of the main limitations of the Walrasian approach when analysing a labour market. Section 3 outlines the motivation underlying Foley's (1994) approach, while section 4 states the formalism of

the general approach. Sections 5 and 6 are the core of the paper. The former depicts the same labour market under both the Walrasian and statistical approaches, while the later compares their results qualitatively. Section 7 concludes.

2. INCONSISTENCIES WITH THE WALRASIAN MECHANISM: TWO EXAMPLES

In this section, we will illustrate the problems that arise when one attempts to confront theoretically these questions, involuntary unemployment and growing income inequality, assuming the Walrasian understanding of how a market functions. We will show how the inability of the theory to explain most of these phenomena as inherent to the market interaction itself, forces the analyst to introduce exogenously other hypothesis in order to match real world facts. This way we aim to point out the inconsistency between some of the most striking social problems nowadays and the dominant theory nowadays to address them.

2.1 Unemployment

As already indicated, the occurrence of involuntary unemployment has been problem largely ignored by classical political economists, coherently with a historical situation where agricultural employment (therefore, self-employment) was the main source of labour, and the growing existence of a ‘reservation army’ guaranteed that industrial wages were close to subsistence levels. It was not until the decade of the 1930’s that the existence of great masses of unemployed workers started to catch the attention of scholar economists, although the main attempt did not come from neoclassical economics, still bewitched from the allocation-related features of the market. It suffices to indicate that by 1932 Lionel Robbins stated his famous definition of economics as ‘the science that studies human behaviour as a relationship between ends and scarce means which have alternative uses’ (Robbins, 2007). The main attempt to think of involuntary unemployment as an inherent problem to the market system has been by John Maynard Keynes’ *The General Theory of Unemployment, Interest and Money* (1936 [1973]). This work’s main conceptual shift was not only to abandon ‘methodological individualism’ (according to which social phenomena are but the unintended outcome of individual actions), but to escape from the Walrasian conceptualization of the auctioneer, and thus from the associated self-regulating process of the market.

Out of the Keynesian tradition, the literature aiming to address the problem of unemployment exemplifies the very deficiencies implicit in the Walrasian device when used as a cornerstone of the theory. On the one hand, we find those approaches that could be considered as *Walrasian*, as they regard prices as an instrument clearing the market and ensuring the attainment of Pareto-efficiency. The RBC literature could be cited as one of its main exponents, in which unemployment is regarded as an optimal response of a representative agent against idiosyncratic and uncertain shocks. Hence, one cannot talk about unemployment either being involuntary or a potential objective for economic policy. On the other hand, there has been a vast literature aiming at conceptualizing it as an internal feature of the market. Therefore, the harmonious conceptualization of general equilibrium is left aside, either by setting wages to be exogenous in an uncertain context (McCall, 1970), by introducing employment lotteries to allocate over an existing population a given amount of labour work (Rogerson, 1988), or by introducing a matching technology within the labour market that creates systemic frictions (Pissarides & Mortensen, 1994). Again, it is necessary to introduce exogenous hypothesis in order to match real-world facts. Therefore, the parallel development of these two branches of economic conceptualization of unemployment stresses the need to leave aside the Walrasian understanding of markets in order to think of problems of such nature, not as a side effect of the theory, but as the very core of it.

2.2 *Income inequality.*

Another striking phenomenon that has caught the attention of economic research in the recent decades is the sharp increase of wage inequality in virtually all of Western societies, both horizontally and vertically. The latter can be easily explained through a Walrasian approach, having different hypothesis been put forward, such as the persistence of a skill-biased technical change, the increasing pressure in the labour markets derived for an increased international competition or the decline of traditional labour market institutions, such as trade unions of minimal wages (especially in the U.S.). This helps to account for the increasing gaps between the lower and upper tails of the income distribution.

However, parallel to this process, Western economies have undergone a process of rising horizontal inequality, that is, individuals who are *ex-ante* identical end up with divergent outcomes after participating in the market. As we have stated before, this is a phenomenon that is implicitly ruled out by the traditional Walrasian mechanism of price

determination. In one hand, we find the tradition started by Truman Bewley (1977), according to which several *ex-ante* identical agents face idiosyncratic shocks, against which they can only partially insure through managing a single stock, such as fiduciary money (Huggett, 1993) or physical capital (Aiyagari, 1994), thus making them *ex-post* heterogeneous. This models can successfully explain the existence of horizontal inequality in a dynamic context, although it is necessary to point out that it does not arise as an implicit result of the way markets function, but due to the existence idiosyncratic shocks in a context of incomplete markets, on which the history of individuals is dependent.

On the other hand, a growing literature assumes the existence of exogenous idiosyncratic shocks in order to explain the rise of both income and consumption inequality throughout the recent period. For instance, some argue that it is the increase in their variance that contributes the most to explain this phenomenon (Blundell & Preston, 1998), while others try to look at the endogenous effects of the organization of financial markets on risk sharing and income inequality (Krueger & Perri, 2006). In sum, *ad hoc* devices must be implemented in order to explain the indisputable existence of phenomena that, according to the sheer core of the theory, are implicitly ruled out.

3. THE STATISTICAL MARKETS APPROACH: CHALLENGING WALRAS

In this paper it is argued that one of the most promising approaches to face these questions is the so-called statistical Equilibrium theory, first developed by Duncan K. Foley (1994). Under this approach, the characterization of the markets as a social institution follows to some extent the Walrasian tradition, in the sense that markets are conceived as ‘arising from the existence of groups of agents who have potentially mutually advantageous trades to make’ (Foley, 1996b). Therefore, Foley (1994) conceives the market as “the process by which individual agents find mutually advantageous production and trading transactions, thereby generating economic activity and at the same time extinguishing some of the transaction opportunities by exploiting them”.

According to this definition, two features stand out. Firstly, agents attend the market voluntarily, having in mind that by doing so they could acquire an individual position they value more, i.e. they act rationally. Markets arise because of their ability to organize a set of individuals in order to better fulfil their needs. Secondly, there is no

sharp distinction between the process of generating opportunities and that of fulfilling them, unlike under the Walrasian approach, where the process of price formation presupposes the opportunities for profit and precedes the realization of the social interaction.

Therefore, the logic underlying the attendance on the side of the agents to the market is remarkably different. Whereas in the Walrasian approach, the pre-determinacy of prices before any actual trade takes place implies that ‘each subsystem (firm or household) deterministically maximizes profit or utility facing uniform prices cried out by an auctioneer’ (Foley, 1996b), so that each agent can perfectly forecast what their state would be after the transactions before they have actually occurred. In this respect, the statistical markets’ approach is radically different. Agents would be attending the market just knowing the set of those transactions they regard as improving their starting position, represented by their offer sets, then would encounter other agents and make transactions in a disorderly and random fashion (Foley, 1994).

This creates an impossibility to achieve the goal of neoclassical economics, that is, to pick one specific transaction arising from the market interaction between agents. However, the aim of the statistical markets theory is not to predict the exact outcome of the market interaction under certain circumstances, but to characterize ‘the most likely statistical distribution of agents over feasible outcomes’ (Foley, 1994). Putting aside the implausible assumption of the auctioneer, prices lose their meaning regarding the optimality of the allocation, although they are still ‘socially meaningful’ prices which emerge from the agents’ interaction, as they order the distribution of agents over actual transaction prices, although they don’t individually inform the economic decisions of an atomistic set of agents.

This setup allows us to overcome some of the problems we have previously identified regarding the Walrasian approach. Firstly, under the Walrasian approach the fact that the determination of the equilibrium prices takes place prior to the actual transactions being realized, implies that agents attending the market with equal endowments and preferences (what we will subsequently refer to as *types*), will leave the market with exactly the same consumption bundle. That is, Walrasian models feature horizontal equality. The statistical approach drastically differs from that. The market interaction will spread the agents of each type over the whole of their offer sets, hence trading at different actual transaction prices. Therefore, this approach shows *horizontal inequality* as an endogenous feature of the way markets function. The

possibility of ending up trading at prices either higher or lower than the average is therefore something inherent to attending the market, not an awkward distortion of it.

Secondly, the role of the Walrasian auctioneer consists in setting a system of prices under which no mutually advantageous opportunities of trade remain feasible after the transactions have taken place. Under the statistical equilibrium this is not true anymore. The fact that the market spreads agents of each type over their entire offer sets implies that the transaction distribution differs from that of the Walrasian prediction (where all agents of each type are concentrated on a single point). Therefore the transaction distribution merely *approximates a Pareto-efficient allocation*, as mutually advantageous transactions will remain unrealized (except in extreme cases). This way, real-world features such as involuntary unemployment are consistent with this way of picturing markets.

4. THE FORMALISM OF STATISTICAL MARKETS THEORY

In what follows, I will proceed to expose the mathematical formalism of the statistical markets approach, as stated in the seminal contribution by Foley (1994, 1996b). We consider a market composed of m commodities. For each agent, a transaction vector $x = (x_1, \dots, x_m) \in \mathbb{R}^m$ expresses the outcome of the market interaction, assuming the components to be a demand if $x_i > 0$ and to be a supply if $x_i < 0$.

We consider that the agents in a market can be organized into r types, each of them being characterized by an *offer set* A^k , which represents the feasible transactions each type regards as improving his position. The offer sets are supposed to contain all relevant information regarding the agent's preferences, expectations, etc. Initially they were considered to be finite by assumption (Foley, 1994), for the sake of simplicity, although "once this point is clear, we see that there is no obstacle to allowing offer sets to contain continua, (...) in this case, the sums in the constraints on the maximum entropy formalism have to be replaced by integrals with respect to these measures" (Foley, 1996b).

The number of agents integrating each type is given by a scalar number n^k , such that $\sum n^k = n$. A *market transaction* $x = (x^1, \dots, x^n) \in \mathbb{R}^m$ specifies the transaction that each individual agent achieves. Feasible market transactions are those under which the market is cleared and, simultaneously, each individual transaction is located within their respective offer sets.

Formally:

$$x_i \in A^{k(i)} \quad \forall i=1, \dots, n$$

$$\sum x_i = 0$$

The set of feasible market transactions is generally very large, so it does not permit to pick one specific market transaction as the outcome of the market interaction, as it has been said before. The approach taken is to assume that in absence of prior information, each market transaction has equal a priori probability of being realized. This might seem quite a conservative approach, although a considerable degree of flexibility can be introduced by managing the relative density of points within each finite offer set (Foley, 1996b). For any market transaction z , we can calculate the *type distribution* of agents over their offer set, as:

$$h_z^k[x] = \{i \mid k(i) = k, x_i = x\} / n^k$$

showing the proportion of agents of type k who achieve a transaction x in the market transaction z . The statistical market approach is concerned with the distribution of agents over singular transactions rather than with one specific market transaction arising from the market. Using this representation it becomes straightforward to formalize an *excess of demand vector* as:

$$\bar{x} = \sum_{i=1}^n x_i \frac{n^k}{n} = \sum_{k=1}^r \frac{n^k}{n} \sum_{A^k} h^k[x]x$$

As agents within each type are economically indistinguishable, different market transactions will yield the same type distribution, hence setting the scene for statistical considerations to be applied. A *market transaction distribution* is the collection of the different type of distributions arising from one specific market transaction, $h = (h^1, \dots, h^r)$. As many different market transactions can give rise to the same market distribution, we can define the *multiplicity* of a type distribution as the number of different market

transactions that produce the same market distribution, which can be calculated through combinatorial methods as:

$$W(h^k) = \frac{n^k}{\prod_{z \in A^k} (n_k h^k(z))!}$$

As the permutation of agents within each type can be done independently of the other types, the *multiplicity of the market distribution* can be easily calculated as:

$$W(h) = \prod_k W(h^k)$$

At this point all elements needed have already been introduced in order to state a new definition of equilibrium. The *statistical market equilibrium* is the market distribution that has the highest multiplicity, that is, the one that can be realized through the largest number of different market transactions. The logarithm of the multiplicity can be approximated by:

$$\ln W(h) = \sum_k n^k S(h^k) \equiv H(h)$$

Where $S(h^k)$ is the *entropy of the type distribution*:

$$S(h^k) = -\sum h^k(z) \ln h^k(z)$$

Therefore, the statistical equilibrium distribution, that is, the distribution with highest multiplicity, can be calculated through what Foley (1994) calls the *Maximum Entropy Program*:

$$\text{Max } H(h)$$

subject to:

$$\sum_x h^k(x) = 1 \quad \forall k$$

$$\sum_{k=1}^r \frac{n^k}{n} \sum_{A^k} h^k[x] x = 0$$

That is, by maximizing the entropy over the type distributions, subject to each type distribution's probability mass being equal to one and the aggregate excess of demand being zero in equilibrium.

The solution to this problem is a well-defined and unique set of Lagrange multipliers, $\mathbf{q} \in \mathbb{R}^m$ which, following the existing literature, will be termed *entropy prices* (for a formal proof of this result see Foley, 1994). These are the social prices emerging from the market. Even though they don't have the same role as prices under the Walrasian approach (where every agent acts as if they could buy as much as they wanted when facing them), they are socially meaningful in the sense that they order the distribution of each type over their offer sets, being the same for all types, that indicates that all the types do belong to the same market.

The solution to this program is of the form of what J. Willard Gibbs called the *canonical form* (Foley, 1994).

$$h^k(x) = \exp(-\mathbf{q}x) / Z^k(\mathbf{q})$$

where $Z^k(\mathbf{q}) = \sum_x \exp(-\mathbf{q}x)$ is called the *partition function* of the type k. Therefore, the market, through maximizing the entropy of the distribution, spreads the agents of each type over the whole of their respective offer sets. The probability of ending up with one specific transaction decreases exponentially with the cost of it. That is why the maximum entropy transaction approximates a Pareto efficient allocation, by concentrating most of the agents of each type at the points with the lowest entropy cost.

When the agents are distributed this way, it becomes straightforward to calculate the excess of demand functions out of the partition functions of each type, as they are just the partial derivatives of the logarithm of the partition function.

$$x^k(\mathbf{q}) = -\frac{\partial \ln Z^k(\mathbf{q})}{\partial \mathbf{q}} = \sum_{x \in A^k} \frac{x \exp(-\mathbf{q}x)}{Z^k(\mathbf{q})}$$

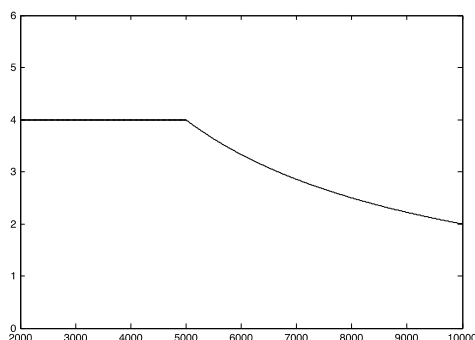
It will be shown below that these expressions are extremely useful. Not only because they allow us to find the statistical equilibrium without having to solve the MEP, but also because there is a clear analogy between these relations and the supply and demand functions under the Walrasian framework. Thus, we will use this similarity in point 7 below.

5. THE STATISTICAL MARKET APPROACH AND A SIMPLE LABOUR MARKET

In the subsequent sections I will proceed to apply the theoretical body of statistical market theory to a simple labour market by re-working the framework firstly advanced by Foley (1996b). We will first start by considering a Walrasian model composed by two types of agents, namely households and firms. Taking this model as a primitive, we will then proceed to ‘translate’ it into the statistical equilibrium framework. Once we have the representation of the same economic environment by the two different traditions, we will compare qualitatively the results yielded from both, thus checking the degree of comparability of both theoretical frameworks.

5.1 The Walrasian Model

We consider a market composed by J firms and by H households, all agents being identical within each type, who interact in a market where they demand and supply labour-power, respectively. On the side of the **demand**, we assume that each firm is endowed with a *variable capital* equal to k , which is the maximum amount of money they can spend in purchasing labour-power. On the other hand, we consider that each unit of labour yields to the firm a constant marginal product of labour (MPL, in what follows). By mere aggregation among firms, we can picture the market demand curve as



Graph 1: Walrasian Demand Curve 1

We can see that the demand curve consists of a horizontal line up to the level $v = \left(\frac{Jk}{mpl} \right)$, to become a downward sloping curve after that point.

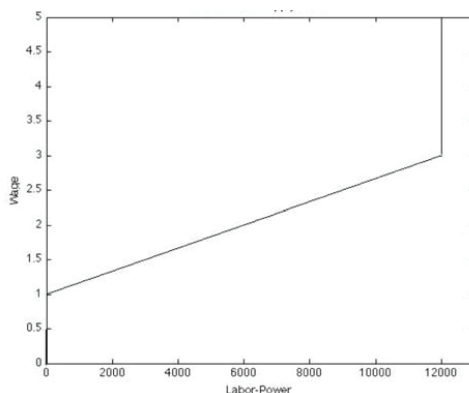
The side of the **supply** of labour will be considered as follows. The market will be integrated by H homogeneous households, where each of them can supply a continuous amount of labour-power to the market, ranging from 0 to 1. Further, there exists a level

of income $W_{\min} > 0$, below which a household will decide voluntarily not to supply any amount labour-power to the market, i.e. W_{\min} will be their ‘reservation wage’. We will consider as well a level of income $W_{\max} > W_{\min}$, such that at that level households will be willing to work ‘full-time’, and above which their labour-power supply will become completely inelastic.

At this point we are considering a decision process more complex than the one considered by Foley (1996b), who merely considers an state of unemployment, at which the household would receive no wage, and a level of income above which the households work full-time inelastically. In our case, by considering the supply of labour to be a continuous function of the income received, we aim to introduce a more realistic process in the determination of wages. Thus, the individual supply of labour we are considering will be of the form:

$$v(y) = \begin{cases} 0 & \text{if } y < W_{\min} \\ \left(\frac{y - W_{\min}}{W_{\max} - W_{\min}} \right) & \text{if } W_{\min} < y < W_{\max} \\ 1 & \text{if } y > W_{\max} \end{cases}$$

Therefore, under our hypothesis, the market supply curve of labour-power would be of the form:

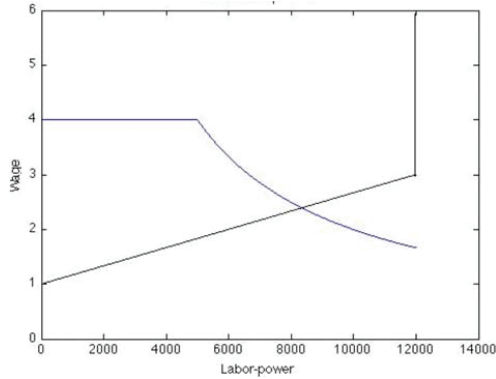


Graph 2: Walrasian Supply Curve

Once we have already depicted both the demand and supply sides, we can easily conceive the Walrasian equilibrium as that level of income at which the aggregate excess of demand of labour is zero, i.e. the level of income at which supply and demand in the market match.

Hence, in our setup, the clearing market condition would be:

$$H \left(\frac{y - W_{\min}}{W_{\max} - W_{\min}} \right) = J \left(\frac{k}{y} \right)$$



Graph 3: Walrasian Equilibrium

These graphs were pictured under the following values: a number of households $H = 12000$, a number of firms $J = 1000$, the marginal product of labor $mpl = 4$, an amount of variable capital for each firm of $k = 20$, and finally $W_{\max} = 3$ and $W_{\min} = 1$. Using these values yields an equilibrium wage of $y^* = 2,39$ and an equilibrium quantity of labour power of $v^* = 8368$.

This conception of the equilibrium yields the features we have already pointed out in previous sections regarding the Walrasian equilibrium. Firstly, under the equilibrium allocation the market yields ‘horizontal equality’, in the sense that all ex-ante identical agents supply exactly the same amount of labour-power, which equals:

$$v^j = \frac{v^*}{H} = \frac{8368}{12000} = 0,697$$

Secondly, it is straightforward to see that it is not possible, under this conception of the market, to talk about involuntary unemployment. Taking the prices as given, each individual finds optimal to supply exactly that amount of labour to the market, according to the individual preferences we have previously stated. Returning to the previous discussion, this phenomenon is due to the specific process of price formation implicit in the Walrasian approach, through which individuals first state their

preferences, then the prices are set to clear the market, and finally all trades take place at the prices previously announced.

5.2 The translation to the statistical markets approach

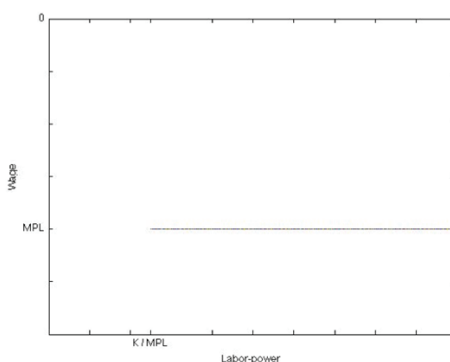
Assuming as a primitive the Walrasian model just exposed, we will proceed now to use that information to construct the model under the statistical equilibrium framework. Under this approach, the elements analogous to the preferences in the Walrasian model are the offer sets. It is clear from the definition of the theory that we are dealing with two types of agents, households and firms, in the sense that their elements are economically distinguishable. Therefore, we will first analyse each type separately, calculating the elements needed to conceptualize the market. Such elements will be their respective offer sets, partition functions and excess of demand functions.

The firms

In the characterization of the firm's offer set we will closely follow the conceptualisation used by Foley (1996b). Again, the offer set represents the set of transactions that each type observes as both feasible and desirable. On the one hand, firms will be willing to spend all their 'variable capital', as there are no further periods that might justify capital accumulation. On the other hand, they will never be willing to pay for a unit of labour-power a price higher than the marginal product they receive. Therefore, their offer set will have the following form:

$$A^f = \left\{ (v, y) \in R^2 \mid y = -k; v \geq \frac{k}{Mpl} \right\}$$

This set is represented in the next figure:



Graph 4: Firms' Offer Set

However, this formulation introduces some novelties with respect to previous one. Contrary to its first formulation by Foley (1994), we are considering an infinite offer set instead of a finite one. This fact does not imply any complication as long as we substitute the sums by integrals. Secondly, this offer set is not bounded, as from the standpoint of a single firm we believe it is not necessary to restrict exogenously the amount of labour-power they might purchase in the market, as long as the number of agents is assumed to be large enough. One alternative to this could be to set a declining density of points (Foley, 1996b). However, this problem is not relevant for our purposes once we integrate over the set in order to get the partition function of the households' type, which has the following form:

$$Z^f(q) = \int_{\frac{k}{Mpl}}^{\infty} e^{-q_v v - q_y (-k)} dv = \frac{e^{k(q_y - \frac{q_v}{Mpl})}}{q_v}$$

The excess of demand functions can be easily calculated by differentiating the logarithm of the partition function with respect to the entropy prices. Hence, the excess of demand of labour-power will be:

$$x_v^f(q) = -\frac{\partial \ln Z(q)}{\partial q_v} = \frac{k}{Mpl} + \frac{1}{q_v}$$

And the excess of demand of income will be:

$$x_y^f(q) = -\frac{\partial \ln Z(q)}{\partial q_y} = -k$$

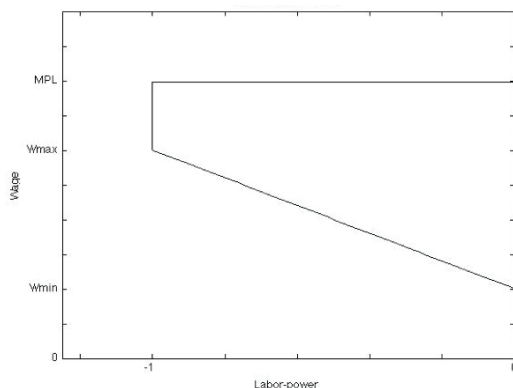
The households

Regarding the characterization of the supply of labour, the distance from Foley's conceptualization of households' preferences is much starker. We consider a continuous labour supply, ranging from 0 to 1, with an associated reservation wage for each level that is linearly increasing with respect to the amount of labour-power traded. For each level of labour-power, we will consider that the individual will accept to trade at any income level weakly higher than the corresponding reservation wage. Apart from that, we consider a range of wages $[0, W_{min}]$ at which the household decides not to supply

any effort, and an upper limit above which the household will voluntarily agree to work full-time.

According to the hypothesis presented so far, the households' offer set does not seem very realistic, as it would imply to be assuming that for each level of labour, each level of income is perceived to have the same probability a priori. There are, in principle, two ways through which we could deal with this problem. On the one hand, we could proceed following the approach used by Foley, that is, exogenously setting a declining density of points on the offer set according to some fixed positive constant (Foley, 1996b). On the other hand, we could set an upper limit to the offer set. Given that the firms will never accept to pay a wage higher than the marginal product they would obtain by employing an additional unit of labour, it seems sensible for the households to include that information among their expectations, and hence to set an upper limit to their offer sets equal to MPL. Therefore, the household's offer set would be the following one:

$$A^h = \{(x, y) \in R^2 : -1 < v < 0, f(x) < y < Mpl: f(x) = W_{\min} - (W_{\max} - W_{\min})x\}$$



Graph 5: Households' Offer Set

This formalization could be modified by introducing a declining density of points in the direction of the perpendicular to the reservation wage relation, but the form of the partition function (which is analogous to the Laplace transform function in physical systems) ensures the results to be well behaved.

Therefore, conditional on this offer set, we can calculate the partition function of the households by just integrating $\exp(-qx)$ over their offer set:

$$Z^h(q) = \int_{-1}^0 \int_{f(v)}^{Mpl} e^{-q_v v - q_y y} dv dy = \frac{e^{-q_y W_{\min}}}{q_y} \left(\frac{e^{(q_v - q_y)(W_{\max} - W_{\min})} - 1}{q_v - q_y(W_{\max} - W_{\min})} \right) - \frac{e^{-q_y Mpl}}{q_y} \left(\frac{e^{q_v} - 1}{q_v} \right)$$

The household's excess of demand functions can be easily calculated by differentiating the partition function with respect to the entropy prices. Hence, the household's excess of demand of labour-power would be given by:

$$x_v^h(q) = -\frac{\partial \ln Z(q)}{\partial q} = \frac{-1}{Z(q)} \left[\frac{e^{-q_y W_{\min}}}{q_y} \left(\frac{e^{q_v - q_y(W_{\max} - W_{\min})} (q_v - q_y(W_{\max} - W_{\min})) - (e^{q_v - q_y(W_{\max} - W_{\min})} - 1)}{((q_v - q_y(W_{\max} - W_{\min})))^2} \right) \right. \\ \left. - \frac{e^{-q_y Mpl}}{q_y} \left(\frac{e^{q_v} q_v (e^{q_v} - 1)}{q_v^2} \right) \right]$$

Analogously, the household's excess of demand of income would be given by:

$$x_y^h(q) = \frac{-1}{Z^h(q)} \left[\left(\frac{e^{q_v - q_y(W_{\max} - W_{\min})} - 1}{q_v - q_y(W_{\max} - W_{\min})} \right) \left(\frac{-w_{\min} e^{-q_y W_{\min}} q_y - e^{-q_y W_{\min}}}{q_y^2} \right) \right. \\ \left. + \frac{e^{-q_y W_{\min}}}{q_y} \left(\frac{-(w_{\max} - w_{\min}) e^{q_v - q_y(W_{\max} - W_{\min})} (q_v - q_y(W_{\max} - W_{\min})) + (w_{\max} - w_{\min})(e^{q_v - q_y(W_{\max} - W_{\min})} - 1)}{(q_v - q_y(W_{\max} - W_{\min}))^2} \right) \right] \\ + \left(\frac{e^{q_v} - 1}{q_v} \right) \left(\frac{e^{-q_y Mpl} (q_y Mpl + 1)}{q_y^2} \right)$$

These are the elements needed to solve the model. It is not necessary to actually solve the Maximum Entropy Program (MEP), as it has a unique solution, and except in degenerate cases (Foley, 1994), its solution is of the form that Gibbs had called canonical. Once the excess of demand functions have been calculated, solving the model implies finding the values of the entropy prices at which the aggregate excess of demand for both commodities is equal to zero. The importance of the entropy prices lies in the fact that they order the distribution of each type over their offer sets.

As it has already been stated in previous sections, the idea of equilibrium under the statistical markets approach is conceptually different. The entropy prices do not represent a constraint at which agents believe that they can trade as much as they want, but they do organize the allocation of transactions among agents. As the proportion of agents ending up in each transaction of their offer set decreases exponentially with the entropy prices, a direct analogy can be drawn with respect to the Walrasian approach in the way prices organize the scarcity in a market.

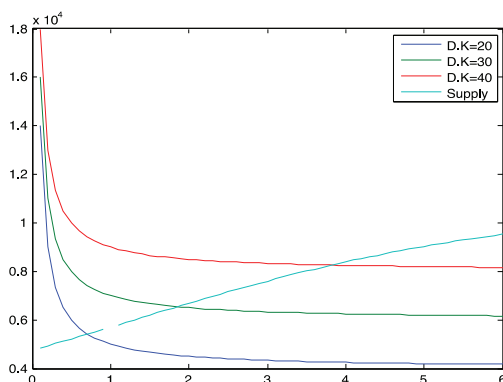
When solving for the equilibrium prices (those at which the aggregate excess of demand of each commodity is equal to zero) we could have aggregated the partition functions of each type into a single one but, by splitting them up, the analogy with a classic supply and demand representation is made clearer.

6. COMPARISON WITH THE WALRASIAN MODEL

The need to take into account the two commodities separately stands in because under the statistical markets approach, unlike under the Walrasian one, the absolute value of the entropy prices has implications for the equilibrium distribution. Therefore, we cannot simply take the price of income as a *numeraire*.

However, what we are ultimately interested in is in analysing the trading of labour-power in the market (that is what a labour market consist of in the end). Hence, we can define the entropy wage as $q_w = q_v / q_y$, and interpret the excess of demand of labour-power function of the firms as a demand in the Walrasian sense, and the excess of demand of labour-power function of the households as a supply function, both as a function of the entropy wage. Therefore, we will focus on each side on the market, and we will analyse how the equilibrium changes according to changes in the variables determining it. At the same time, we will try to analyse the relation of the equilibrium in this labour market with the variables underlying it, and to what extent the logic underlying it resembles that of the Walrasian approach. In the graphs referring to the statistical approach, we will display the entropy wage, i.e. the prices, in the horizontal axis, to stress the fact that the excess of demand relations are a function of it. In order to make things easier to the reader, with regards to the graphs corresponding to the Walrasian approach we will follow the convention of displaying the prices in the vertical axis.

In the first place, we will plot the equilibrium regarding the labour-power commodity, varying the level of **variable capital** each firm is endowed with in order to purchase labour-power. We show in the following graph how the excess of demand in labour-power of the firms, that is, the demand of labour in this market, varies for different values of variable capital, as functions of the entropy wage:



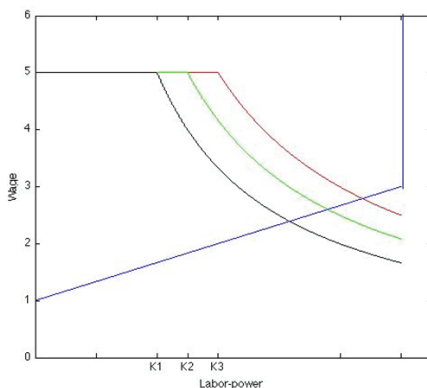
Graph 6: Statistical Equilibrium under different stocks of Variable Capital

We plot the equilibria corresponding to levels of variable capital of $k=\{20,30,40\}$. These relations are plotted for values of $Mpl=5$, $J=1000$, $H=12000$, $W_{max}=3$ and $W_{min}=1$.

We see that both the entropy wage and the amount of labour-power traded are positively related to the level of variable capital firms are endowed with. The supply of labour remains unchanged, as the excess of demand of labour-power of the households is not a function of the amount of variable capital. As the firms enjoy a higher amount of money to be spent in labour, while keeping the supply constant, the demand for labour is affected by upward displacements, therefore univocally causing an increase in the amount of labour traded and of its price.

By doing an analogous experiment under the Walrasian approach, we obtain qualitatively the same results. The following graph is pictured under the same values stated before, with the only difference that the values given for the level of capital are now $k=\{k1=20, k2=25, k3=30\}$.

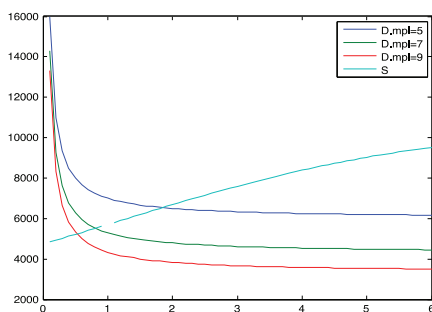
Both models yield the same prediction under this scenario. The higher the amount of capital each firm is endowed with, the higher will be the level of employment and, given that the reservation wage of the households is increasing with respect to their level of income, the higher the equilibrium price will be:



Graph 7: Walrasian Equilibrium under different stocks of Variable Capital

Hence, in this case we can say that the statistical market results are well-behaved with respect to the level of variable capital.

According to our characterization of the types, the remaining variable affecting the decisions on the side of the firms is the benefit obtained per unit of labour, i.e. the **marginal productivity of labour** (Mpl). We will proceed to evaluate qualitatively the variations in the equilibrium depending on how productive the workers are. As both the demand and the supply are dependent on it, we will split the analysis into two parts. First, we will observe how it affects the decision of the firms, who are most importantly affected by this variable, and then we will see how the changes induced in both types interact. We show in the following graph how demand for labour varies after changes in the productivity of labour, as a function of the entropy wage:

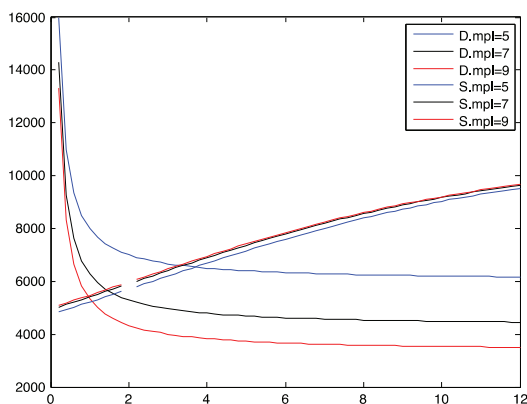


Graph 8: Statistical Equilibrium Demands under different values of MPL

We plot the equilibria corresponding to levels of variable capital of $Mpl=\{5,7,9\}$. These relations are plotted for values of $k=30$, $J=1000$, $H=12000$, $W_{max}=3$ and $W_{min}=1$. We see that the less productive the workers are in their jobs, the higher the demand of labour for each level of the entropy wage is. Hence, keeping the supply constant, given its upward slope, the productivity of workers is inversely correlated with the equilibrium entropy wage and the total level of employment.

The rationale underlying this result is to be interpreted as follows. If the productivity yielded by each worker is lower, then for a given level of output (which implicitly we are considering exogenous) there should be a higher aggregate level of employment. The total amount of workers being constant, given that they show an increasing reservation wage in the amount of labour each of them supplies to the market, a higher level of labour traded in the market can only be achieved by an increase in the price of labour (the entropy wage in our case).

When considering the changes induced in the supply of labour (the households) as well, the equilibrium is not significantly altered with respect to the case shown before. As before, we plot the equilibria corresponding to levels of variable capital of $Mpl=\{5,7,9\}$, but now varying both excess of demand relations. These relations are plotted for values of $k=30$, $J=1000$, $H=12000$, $W_{max}=3$ and $W_{min}=1$:



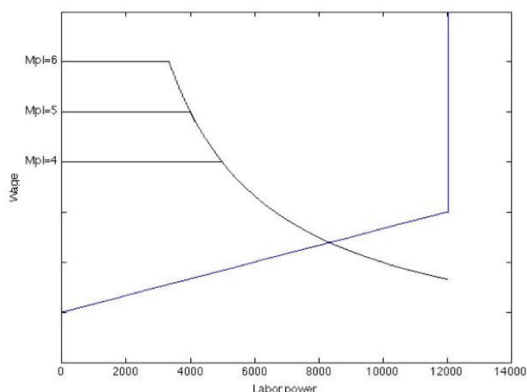
Graph 9: Statistical Equilibrium under different values of MPL

Even by changing the values of the productivity of labour in a range that would completely pervert the side of the demand, the changes in the supply of labour will keep on being almost imperceptible. The only important lesson that we learn from doing this is that in the cases where $Mpl < W_{max}$, the supply of labour would stop showing an

upward slope in all of its domain, from being almost flat at the beginning to show a downward slope after a level of the entropy wage.

The effects under the Walrasian approach are ambiguous. As long as the part of the demand curve intersecting is the one corresponding to the hyperbola, the equilibrium would not be altered by the marginal productivity of the firms. That is because at that point the firms are paying a price lower than the marginal product they receive, but buying a higher amount of labour-power. At those levels of employment, it is not relevant for the firms how productive the workers originally were.

That situation is shown in the following graph:



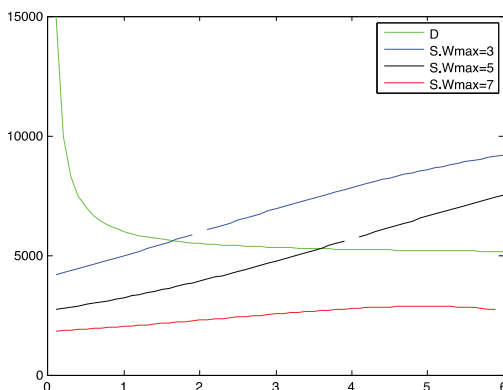
Graph 10: Walrasian Equilibrium under different values for MPL

It becomes imperative to stress that, whereas under the Walrasian equilibrium all employed workers would be receiving exactly the same wage, under the statistical approach it comes as an implicit result that some workers will receive very high wages and others a much lower one. In a situation where the equilibrium wage varies according to the amount of labour traded it seems intuitive to introduce some randomness in the individual outcomes from the market. This uncertainty is based on a lack of information in both sides regarding their environment, so that their very interaction fails to achieve the degree of synchronization present in the Walrasian understanding of markets. This is the actual meaning of the entropy prices. The uncertainty being an endogenous result of the market, the prices (and hence the forces underlying them) indicate the pressure effected on that uncertainty, modelling the distribution over outcomes, but in general not turning it into a single point.

So far, we have focused on how the demand for labour is affected by changes in the variables it depends upon. Now it is time to focus on the behaviour of the households. We will do that by investigating how the **changes in the reservation wage relation** of

the households alter the equilibrium configuration. As the individual supply of labour increases linearly with respect to the level of income, we will proceed to do this by changing the interval $\{W_{min}, W_{max}\}$, and hence the labour-leisure preference relation of the households. In the first place, we do this by keeping constant the level at which households voluntarily decide not to supply any work to the market (that is, W_{min}), while giving different values to the level at which they decide to work full time.

We plot the equilibria corresponding to the values of $W_{max}=\{3, 5, 7\}$, this way changing the preferences of the households and their offer sets. These relations are plotted for values of $Mpl=6, k=30, J=1000, H=12000$ and $W_{min}=1$:

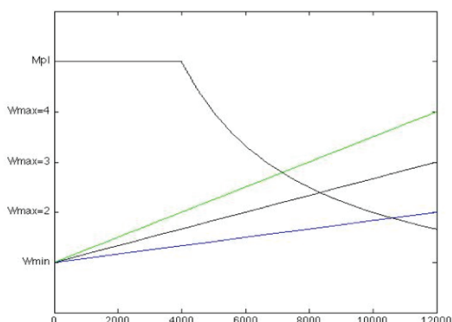


Graph 11: Statistical Equilibrium under different values for Wmax

We see that the higher it is the value of W_{max} , the lower the supply of labour is. Expanding the level of income at which the individuals decide to work full-time means that for each level of the entropy wage, q_w , each individual will be willing to supply a smaller amount of work to the market. That means that the amount of labour traded in equilibrium will be lower, and its price will be higher. Therefore in the aggregate, for each level of the entropy wage, the total supply of work will be lower the higher we set W_{max} . This result perfectly matches the Walrasian logic under which the total amount of work supplied to the market decreases as long as the individuals are discouraged to supply their workforce to the market.

On the other hand, this graphs allows us to see the pervasive effects caused in the supply of labour by setting values such that $Mpl < W_{max}$, as it can be seen in the case where $W_{max}=7$. In that environment it is not even possible to find an equilibrium allocation where supply and demand would match.

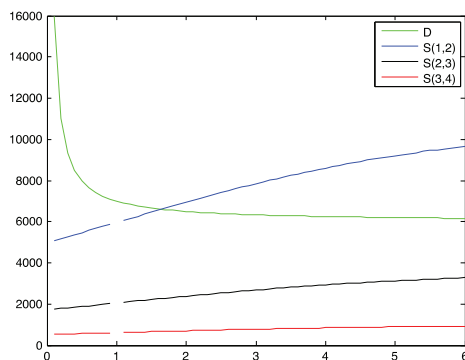
If we try to do the same exercise under the Walrasian framework, we would obtain qualitatively the same result. We show in the following graph how the equilibrium would have changed for a range of values of $W_{max}=\{3, 5, 7\}$. The rest of parameters are given the same values than before:



Graph 12: Walrasian Equilibrium under different values for W_{max}

The logic operating here is the same. The higher W_{max} is, the less effort each individual will be willing to provide for each level of income. This results in a lower market supply curve, and hence into an equilibrium allocation featuring a higher wage and a lower amount of labour eventually traded.

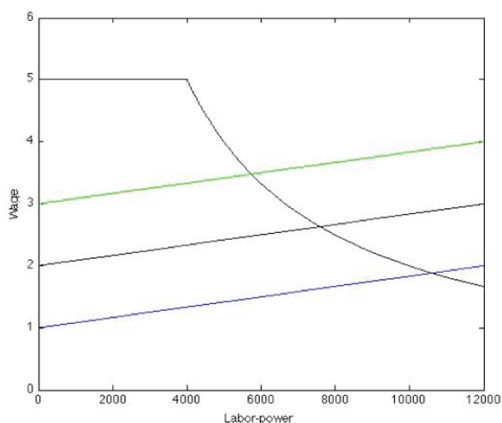
Following a similar logic, we explore what would occur if we maintain the gap $\{W_{min}, W_{max}\}$ constant, but its absolute values differ in the distance separating them from the marginal product of labour the firms obtain by employing them. The next graph is plotted for levels of the gap $\{W_{min}, W_{max}\}=\{1-2, 2-3, 3-4\}$, hence keeping the distance between the two terms constant. These relations are plotted for values of $Mpl=6, k=30, J=1000, H=12000$ and $W_{min}=1$:



Graph 13: Statistical Equilibrium under different values for $[W_{min} - W_{max}]$

The results shown clearly match our intuition that, if the households were to require a higher minimum wage to start supplying some labour-power to the market, for each level of the entropy wage the total supply of the market would have been lower. Indeed, we see that for the intervals (2,3) and (3,4) it is not even possible to get an equilibrium allocation of this commodity under the present parameterization.

Again, by doing the same experiment under the Walrasian approach we find the same prediction. The higher the values defining the interval are, the lower the supply of labour will be for any given value of the entropy wage, therefore causing the equilibrium to feature a higher wage and a lower amount of labour traded. This result is analogous to the one found under the statistical equilibrium approach.



Graph 14: Walrasian Equilibrium under different values for [Wmin - Wmax]

Lastly, we would like to analyse the existence of **involuntary unemployment** in this model. The characterization of the market in Foley's (1996b) paper makes it easier to deal with it, as he considers a state where the household supplies no work and receives no income, and the rest of them where the household works full-time and receives varying levels of wage in compensation. Hence, as the households either work full-time or they don't work, he interprets the ratio of unemployment in the economy as the probability for a household of ending up in the first state.

The higher complexity of the working decisions in our model creates some problems when we try to think about the possibility of unemployment. We can observe the total amount of labour traded in the market, and we know that agents will supply different amounts of labour-power (indeed, the formalism of the statistical markets approach allows us to be sure that the households will be spread all over their offer set),

although by just looking at the labour-power excess of demand function, we cannot know what is the exact amount of labour that each specific agent is supplying.

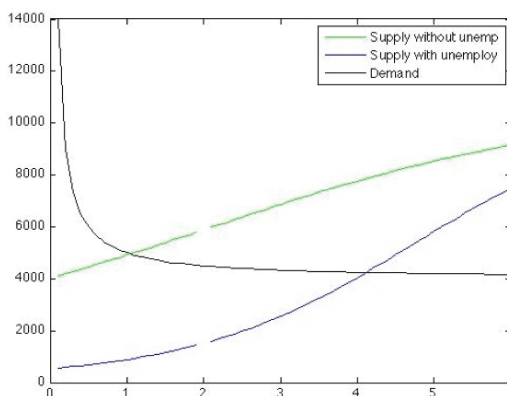
We will proceed the following way: we will add an additional state in the households' offer set where they don't receive any income nor they supply any labour to the market. Foley does this in a finite set, so that the probability assigned to that state would be depending on the probabilities corresponding to the rest of the set. As we are operating within a continuous set, we can set the probability assigned to that state exogenously by determining its mass. Hence, the partition function of the households would be modified in the following way

$$\hat{Z}^h(q) = du + Z^h(q)$$

Under this setup, we would consider that the proportion of unemployed workers is:

$$u = \frac{du}{Z^h(q)}$$

For simplicity, in what follows we will assume that $du=1$. We plot in the following graph the supplies of labour corresponding to both partition functions, with and without an explicit unemployment state. These relations are plotted for values of $Mpl=5$, $k=30$, $J=1000$, $H=12000$ and $W_{max}=3$ $W_{min}=1$.



Graph 15: Statistical Equilibrium with and without and Unemployment State

We see that by explicitly introducing a state of unemployment with no income, the amount of labour traded in equilibrium decreases and its price is increased. Given that the households will be spread all over their offer sets, the total amount of labour traded

will be lower, *ceteris paribus*, and its price will be higher as there is the same amount of money-income to be allocated among a smaller number of agents. This implies that those who end up working will be better-off under the existence of the state of unemployment, and therefore we can conclude that the unemployment state will be by necessity involuntary.

7. CONCLUSION AND FINAL REMARKS

Arguably, the predominance of the neoclassical paradigm after the decade of 1870 is due to the solution it offers to several problems related with the determination of prices, implicit in the labour theory of value present in the works of the classical economists. In spite of the advantages it offers, the awkwardness of the Walrasian mechanism of price determination affects the validity of the predictions it yields. Some basic results of the neoclassical paradigm, such as the first and second welfare theorems, rest to a very large extent upon the Walrasian fiction of the auctioneer.

Therefore, we face nowadays an enormous amount of normative judgements that critically depend on the assumptions adopted regarding the functioning of markets. As it has been argued so far, it becomes necessary to discuss the very core of the theory in order to re-evaluate the accuracy of a number of policy recommendations and normative propositions. By assuming that the transaction prices are determined before any sort of interaction occurs (instead of the latter being the process itself by which prices emerge), we face two main problems. On the one hand, we have to decide whether the Walrasian mechanism actually resembles the way markets operate, and on the other, it becomes very complicated to critically appraise certain phenomena we observe in real-world markets.

The statistical markets approach offers the possibility of overcoming some of these issues. By challenging the definition of the market as a social institution, the motivation underlying the attendance of the agents to the market or the way prices emerge from the social interaction as well as its meaning, we aim to show how some real-world phenomena, such as the inability to achieve a fully Pareto-efficient allocation or the existence of horizontal inequality, can be explained as implicit features of the market mechanism. Obviously, this is done at a cost. It is necessary to face the question of whether both methods are commensurable, that is, whether we can compare both to determine which one shows a higher degree of accuracy.

In order to answer these questions, we focused on a simple labour market, inspired by the one depicted by Foley (1996b). In the first place, we wanted to enhance the model by increasing the complexity of the characterization of the agents, especially on the side of the households. This way, we aim to introduce a higher degree of realism in the model, by making the outcome be determined by a higher number of variables. Secondly, by introducing these changes, it becomes possible to proceed to check the extent to which both approaches are commensurable. Having used the Walrasian model as a primitive to build the statistical model, we ensure a degree of comparability between the two that would not have been achieved otherwise. We saw how the evaluation of different changes in the environment by the two approaches was highly similar. This proof of the statistical model being well behaved, in this very specific sense, does allow us to pay higher attention to those features which mean, in our opinion, an advancement with respect to the Walrasian approach. By this we mean the possibility of endogenously producing horizontal inequality, stressing the non-predictability of the market, as well as understanding the concept of Pareto-efficiency as a direction towards which the market moves, rather not as a characteristic feature of it.

Our discussion of these issues suggests several directions where further research could be conducted. Firstly, different specifications could be tried, such as introducing a more complex one in the side of the firms. This way a higher degree of realism would have been achieved, as by introducing a higher amount of variables determining both types in the market the robustness of the results presented so far could be increased. Secondly, it would be a very rewarding experiment to analyse in this model the changes in the dispersion of the distribution resulting from modifications in the parameters determining the model. The matrix of variances and covariances of the distribution of each type across their offer set would correspond to the second derivatives of the logarithm of the partition function (Witte, 2013). Lastly, we have pointed out how the simplicity of Foley's characterization of the households' offer set allows him to study the rate of unemployment in this economy. However, in our case, the higher complexity of our modelling does not enable us to accomplish such a straightforward appraisal. In our framework we have exogenously determined, for the sake of simplicity, the probability assigned in the offer set to the unemployment state. Under this approach, it would be very rewarding as well to accomplish an empirical investigation trying to determine a sensible range of values for this parameter.

Finally, when interpreting these results, it is important not to forget that the main theoretical shift is introduced at the very core of the theory, shattering its most inner hypothesis. A different conceptualization of the market as a social construction, the motivation underlying the attendance of agents to the market, the random and disordered actualization of the economic possibilities or the informational structure of the individuals calls the attention to the relevance of the core hypothesis of neoclassical economics. They might be taken for granted to such a large extent that we might end up forgetting about them when we try to answer some economic question.

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