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Olga Alonso-Villar
Universidade de Vigo
Dpto. de Economía Aplicada
Apto. 874, 36200 Vigo – Spain
Tel.: (34) 986 81 25 07
Fax: (34) 986 81 24 01
e-mail: ovillar@uvigo.es

SPATIAL DISTRIBUTION OF PRODUCTION AND EDUCATION

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Abstract

There is no doubt that people like to migrate to large cities because they can acquire a wider range of products and jobs, but also because they can exchange information and ideas in an easier way. In this respect, we will attempt to explain the formation of metropolitan areas through a general equilibrium model in which concentration emerges not only from the interaction between increasing returns to scale at the firm level, transport costs and the mobility of labor, but also from human capital externalities. Our aim is to underline the role of human capital as a factor that fosters both the agglomeration of the economic activity and cities' growth. The paper shows that there is new scope for government activities.

JEL classification: D43; D62; R12; I28

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1. Introduction

As Krugman (1996) points out, any model of economic geography must display the tension between centripetal forces, which favor agglomeration, and centrifugal forces, which limit the growth of such agglomerations. In Krugman's models,[1] centripetal forces arise from the interaction between increasing returns to scale at the firm level, transport costs and the mobility of the industrial labor force. Increasing returns to scale imply that the production of each good will take place in a single location. The existence of transport costs implies that the best locations for a firm will be those with easy access to markets, and the best locations for consumers, those with easy access to goods. Thus, concentration is the result of a self–reinforcing process of agglomeration.

However, as Lucas (1988, p.38) says: "Of course, people like to live near shopping and shops need to be located near their customers, but circular considerations of this kind explain only shopping centers, not cities." He suggests that the central role of cities, and therefore urban agglomeration, is concerned with the external effects of human capital. Talks turn information into productive knowledge and cities are precisely the places where the cost of collecting and spreading information is lower. Thus, it seems plausible that such externalities are strongest within a city. Some empirical works sustain this hypothesis (Rauch, 1993; Glaeser et al. 1992). Therefore, the external effects of knowledge that have been emphasized by growth theorists (Romer, 1986; Lucas, 1988) can also explain the growth of cities. As a matter of fact, studying English cities from 1861 to 1961, Simon and Nardinelli (1996) found that those cities with high proportions of business professionals (information–orientated professionals) grew more rapidly.

There is no doubt that people like to migrate to large cities because they can acquire a wider range of products and jobs (as in Krugman's models), but also because they can exchange information and ideas in an easier way. In this respect, we will attempt to explain the formation of metropolitan areas through a general equilibrium model in which concentration emerges not only from the interaction between increasing returns to scale

[1] See, for example, Krugman (1991, 1993a, b).

at the firm level, transport costs and the mobility of labor, but also from human capital externalities. Our aim is to underline the role of human capital as a factor that fosters both the agglomeration of the economic activity and cities' growth, in a sense that will be explained later, in the context of a core–periphery model *à la* Krugman.[2]

Therefore, in this model there are not only pecuniary urbanization externalities (which explain why firms want to locate where local demand is high), but also knowledge spillovers. As Fujita and Thisse (1996, p. 346) suggest, "an economic agglomeration is created through both technological and pecuniary externalities, often working together." By combining technological with pecuniary externalities, the paper combines the neoclassical urban approach[3] and the new economic geography.[4]

In contrast with Krugman (1991), in this paper centrifugal forces arise from congestion costs, rather than farmers. Nowadays, farmers by their sheer number, seem not to be the economic force halting the growth of cities. Much more compelling is the fact that large cities suffer from urban traffic problems, pollution and high housing prices that make small cities relatively more attractive places to live in.

By comparing this model with Krugman's models, we can now see that a new stable equilibrium configuration emerges: the coexistence of two cities of different size, one large and the other small (a typical metropolitan configuration). This makes the model more realistic.

Finally, we have included the government as one more economic agent with the ability to modify investments in education and, therefore, internalize the externality that human capital produces in the economy. The paper shows that there is new scope for government activities. In fact, it suggests that there is a trade–off between the positive externalities of knowledge that an increase in education produces and the negative congestion externalities

[2] We are not interested in explaining the links between residential location, investment in education and production inside a city, like Benabou (1993), nor studying segregation (like Abdel–Rahman and Wang, 1997).

[3] See Henderson (1974), for instance.

[4] See Krugman (1996) for more details.

derived from the high level of agglomeration associated with high levels of human capital, implying that the allocation of labor across cities may be inefficient. On the other hand, not only history affects spatial configurations, but also the government can do so by choosing the level of investments in education. Some results on the importance of the timing of these investments are presented.

The paper is organized as follows. In section 2 we present the assumptions of the model and analyze the short–run equilibrium. In section 3, we study the long–run equilibrium. The role of the government as an agent internalizing the externality that education involves is described in section 4. Comparisons of different long–run equilibria are shown in the same section. Finally, section 5 concludes.

2. The basic model

2.1 Assumptions of the model

We assume an economy with a large number of potential goods that appear in the utility function in a symmetric way. All consumers have the same CES tastes:

$$U = \left(\sum_i c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where c_i is the consumption of good i and the elasticity of substitution between any two goods, σ, is greater than 1.

There are two types of workers in this economy: high–skilled workers and low–skilled workers. All individuals have one unit of time. As in Lucas (1988) a worker with a high human capital level is the productive equivalent to several workers of a lower human capital level. In fact, a high–skilled worker is equivalent to π low–skilled workers, where $\pi > 1$. Firms compete in a monopolistic regime of the Dixit and Stiglitz (1977) type, where labor is the only factor of production. All goods are produced under economies of scale with the same technology

$$L_{ij} = \alpha(k_j^h) + \beta(k_j^h) x_{ij}, \quad (\alpha > 0, \beta > 0) \tag{2}$$

where L_{ij} denotes the units of effective labor force (high–skilled labor) needed to produce x_{ij} units of good i in city j, and k_j^h denotes the human capital level (the number of high–skilled workers) of that city. We assume, as empirical evidence supports (Rauch, 1993, p. 399), "productivity benefits from geographic concentration of human capital caused by sharing of ideas." We capture the essence of this effect by considering

$$\alpha(k_j^h) = \frac{\alpha}{(1+k_j^h)}, \tag{3}$$

$$\beta(k_j^h) = \frac{\beta}{(1+k_j^h)}. \tag{4}$$

Since both functions are monotonically decreasing in their arguments, the higher the level of human capital, the higher the productivity of all individuals living in the same city. Hence, these non–constant functions lead to the existence of a positive externality derived from education.

Two elements affect the level of human capital in a city: its average level and its absolute value. The former implies that the higher the average level of human capital (the proportion of high–skilled workers) in a city, the more rapid the diffusion of knowledge because of the interactions between high and low–skilled workers. The latter means that the higher the number of high–skilled workers in a city, the more rapid the growth of knowledge because of the interactions among the high–skilled workers.

We also assume that a low–skilled worker may become a high–skilled worker through education. In order to acquire education, low skilled–workers must pay a price, P^e, to high–skilled workers, who are the educators in our model.[5] Furthermore, both educators and pupils lose a portion of their time due to this formation process. Education implies two losses. A low–skilled worker loses c_1 units of his time when he wants to become a high–skilled worker, while each high–skilled worker spends c_2 units of his time for every low–skilled worker that he decides to educate. Thus, both agents have to decide how much of their time they want to allocate to work and how much to education.

[5] A similar assumption can be found in Upadhyaya (1994), where skilled workers produce goods and educate the unskilled ones and in Zhang (1996) where the old agents spend some time in production and some other in teaching their children.

In this model, there are other costs due to the transportation of goods between cities and due to the congestion cost experienced within cities. These costs take the usual *iceberg* form: a proportion of the good produced by a firm melts before it reaches consumers. On the one hand, when one unit of a good is transported from the city where that good is produced, j, to the city where the consumer is, k, the amount that arrives is only $e^{-\tau D_{jk}}$, τ being the transportation parameter, and D_{jk} the distance between the cities j and k. On the other hand, inside every city there are some negative elements such as urban transportation, housing prices, or environmental pollution, which make the larger cities places not attractive to live in. We include all these negative factors under the term of *congestion costs*. So when one unit of a good is produced in, or arrives at, city k, any consumer living in that city can obtain only a proportion $e^{-\gamma\lambda_k}$ of the good due to congestion costs, where λ_k is the population living in city k and γ is the congestion parameter.[6] We can see that the proportion melted away is greater, the higher the population size of the city. Hence, we have that when the good is produced in the city where the consumer is living, there is only the loss due to congestion costs. However, if the good is shipped from another city, we must consider two losses jointly: one derived from transportation between cities and another from the congestion cost experienced by consumers in the city where they live.[7]

Finally, we assume that in the long run individuals move toward locations with higher real wages.

2.2. The short–run equilibrium

The model we discuss here is one of monopolistic competition, where each firm produces only one good under increasing returns to scale. Increasing returns imply that every good is produced in only one location and, therefore, different cities have different baskets of

[6] We assume that total population is normalized to 1.

[7] We could treat intra–urban congestion in a more explicit way, such as land consumption and/or traffic congestion in cities. But, such an extension would not significantly change the main conclusions of this paper. Therefore, we take the simplest form of urban congestion.

goods. Since there is love–for–variety, individuals demand not only the goods produced in their own city but also goods from other cities. The price that every consumer will finally pay for every unit of good consumed (c.i.f. price) will depend on both the price charged by the firm (f.o.b. price) and the transport and congestion costs. Hence, every firm faces two different demand functions: the demand by individuals living in the place where the firm is located (interior demand) and the demand by individuals living outside (exterior demand). Because both demands have the same price elasticity, σ, congestion and transport costs do not affect the behavior of firms. Then it can be shown that the f.o.b. price charged by the firm that produces good i in city j is

$$p_{ij} = W_j \frac{\beta}{1 + k_j^h} \frac{\sigma}{\sigma - 1}. \tag{5}$$

We can see that this price (which is a constant mark–up over marginal cost) only depends on the wage rate (per unit of effective labor), W_j, and the level of human capital. The other parameters are common for all locations. Therefore, all goods produced in the same city have the same price. Because of the relationship between high and low–skilled workers, the wage rate of a low–skilled worker is equal to that of a high–skilled worker of his city divided by π.

Monopolistic competition means that firms enter the market until profits become zero. This implies that the amount of good i produced by a firm located in city j is

$$x_{ij} = \alpha(\frac{\sigma - 1}{\beta}). \tag{6}$$

Since $\alpha(.)$ and $\beta(.)$ depend on human capital in the same way we have that the amount produced by any firm does not depend on specific characteristics of population living in the city where the firm is located.[8] As every firm inside city j produces the same amount and has the same technology, the number of firms being there, n_j, will depend on the number of high–skilled and low–skilled workers living in that city. This number can be obtained by dividing the number of units of effective labor in city j by the number of units of effective labor that a firm needs.

[8] This will facilitate the calculation of the optimal level of education, as we will see later.

Therefore, ceteris paribus, the higher the level of human capital in a city, the higher the number of firms (and goods) that can be supported there. In other words, these knowledge spillovers explain why cities grow. Hence, the number of varieties and firms in the economy is not fixed, as in Krugman (1991) among others, but it depends on its human capital level. We assume that the human capital level of any city can be changed if low–skill workers acquire education. By investing in education, the number of high–skilled workers increases and, therefore, the level of human capital increases. We will calculate the price of education, P_j^e, and the amount of education in equilibrium on what follows.

The income level of a low–skill worker in city j is $\frac{W_j}{\pi}$ if he does not get an education and $(1-c_1)W_j - P_j^e$ otherwise, because he has lost c_1 units of time in the process of education and had to pay a price for it. Hence, this individual will acquire an education if and only if:

$$P_j^e \leq W_j(1 - c_1 - \frac{1}{\pi}). \tag{7}$$

We denote the right hand side, $W_j(1-c_1-\frac{1}{\pi})$, by P_j^{e*}.

Analogously, the income level of a high–skilled worker in city j if he does not educate anybody is W_j and if he educates m_j low–skilled individuals is $(1-c_2 m_j)W_j + P_j^e m_j$. Therefore, this individual wants to educate these low–skilled workers if and only if

$$P_j^e \geq c_2 W_j. \tag{8}$$

We denote the right hand side by P_j^{e**}.

In order to obtain the supply function, we must take into account that the maximum number of low–skilled individuals that any high–skilled worker can educate is $\frac{1}{c_2}$, and therefore, the maximum number of those who can receive education is $\frac{\lambda_j^h}{c_2}$, where λ_j^h is the number of high–skilled workers in city j.

To calculate the amount of education in equilibrium, we must distinguish three cases:

- $P_j^{e*} > P_j^{e**} \Longleftrightarrow c_1 + c_2 < 1 - \frac{1}{\pi}$

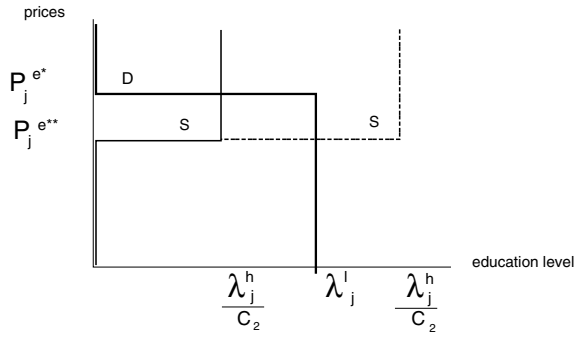


Fig. 1

In this case we can see that the maximum level of education is offered. Then either all low–skilled workers, λ_j^l, can become educated (if $\frac{\lambda_j^h}{c_2} > \lambda_j^l$) or only some of them, $\frac{\lambda_j^h}{c_2}$, otherwise. Moreover, the price of education coincides with P_j^{e**} in the former and with P_j^{e*} in the latter.

- $P_j^{e*} < P_j^{e**} \Longleftrightarrow c_1 + c_2 > 1 - \frac{1}{\pi}$.

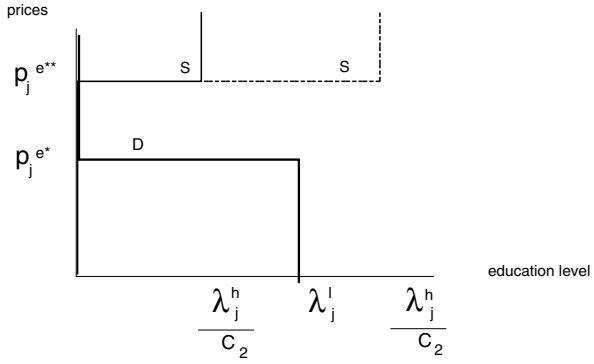


Fig. 2

Then, nobody receives education in equilibrium since the maximum price that low–skilled workers are willing to pay for education is lower than the minimum price that high–skilled workers are willing to accept.

- $P_j^{e*} = P_j^{e**} \Longleftrightarrow c_1 + c_2 = 1 - \frac{1}{a}$.

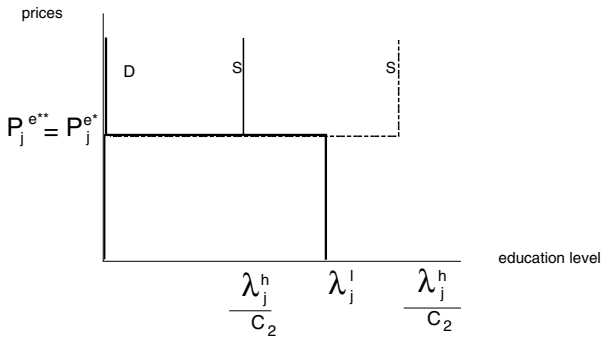


Fig. 3

In this case, every amount of education between 0 and $\frac{\lambda_j^h}{c2}$ with price $P_j^e = P_j^{e*} = P_j^{e**}$ are equilibria.

Given the population of every city k, λ_k, we will calculate now the wage rate of its high–skilled workers, W_k. From now on, we will consider that the wage rate in city 1, W_1, is equal to 1. We will also choose the units in which goods are measured in such a way that $\beta = \frac{(\sigma-1)}{\sigma}$. Therefore, the f.o.b. price charged by a firm takes the simplest form

$$p_{ij} = \frac{W_j}{1 + k_j^h}, \tag{9}$$

where $k_j^h = \lambda_j^h + \lambda_j^h m_j$ is the human capital accumulated.

Given the number of high and low–skilled workers in every city (the human capital level), the congestion (γ) and transportation (τ) parameters, and the taste for variety (σ), we can calculate the equilibrium wage rates in every city and their price indices by solving the following system of equations:[9]

$$W_j = \left[(1 + k_j^h)^{\sigma} \sum_k Y_k (e^{-(\tau D_{jk} + \gamma \lambda_k)} T_k)^{\sigma - 1} \right]^{\frac{1}{\sigma}} \tag{10}$$

[9] See Appendix.

$$\left.\begin{aligned} T_j &= \left[\sum_k \overline{\lambda}_k (1+k_k^h)^{\sigma} (W_k e^{\tau D_{kj}+\gamma\lambda_j})^{1-\sigma}\right]^{\frac{1}{1-\sigma}} && (11)\\ Y_j &= \overline{\lambda}_j W_j && (12)\\ k_j^h &= \lambda_j^h + \lambda_j^h m_j && (13)\\ \overline{\lambda}_j &= \lambda_j^h(1-c_2 m_j) + (\lambda_k^l - m_j\lambda_j^h)\frac{1}{\pi} + m_j\lambda_j^h(1-c_1), && (14) \end{aligned}\right.$$

where T_j is the price index, Y_j is the income and $\overline{\lambda}_j$ denotes the units of effective labor available for production in city j.

This system of equations can be analytically solved in some simple cases, for example, when the population is concentrated in only one location or when each location has the same size. Otherwise, we will need the use of computers to solve the system for some given values of the parameters.

3. The long–run equilibrium

In section 2, we calculated the equilibrium of the economy for a given distribution of the population. In this section, we assume that there are only two cities in the economy and that individuals can move across them. Let us consider the distance between the two locations as 1. For analytical convenience, we assume that human capital is initially evenly distributed between the two cities in such a way that, in any of them, the proportion (not the number) of high–skilled workers is the same.[10] This implies that there is no difference among the two cities in the relative education level offered in equilibrium, which makes the model easier to work with. This does not mean that the number of new high–skilled workers is going to be the same in any location, but it is the probability of being educated and the education price which is independent of location. Actually, in the previous section we obtained that the probability of acquiring an education is the same in any city (since the teacher–pupil ratio is independent of location) and the price of education is a proportion

[10] This common average level of education in the two cities could be justified by thinking that new generations are obliged to acquire a certain level of education by law (high skill) while older generations had a lower level of skill. Old and young are uniformely distributed on the space.

of the wage rate, a ratio that is also the same at each location. Therefore, the only difference between both cities is the real wage workers can obtain in each of them. Cities do not differ with respect to the relative education they can offer, even though they do not have the same absolute human capital level and, hence, they can offer different absolute levels of education.

Let the real wage be the ratio between the wage rate and the price index, namely, $\omega_j = W_j T_j^{-1}$. We now suppose that individuals move toward the city that offers them higher real wages and that the composition of the flow of emigrants is kept as constant, in order to simplify the analysis. Define a (long–run) equilibrium as any distribution of population between the two locations such that $\omega_1 = \omega_2$, or $\omega_1 < \omega_2$ ($\omega_1 > \omega_2$) and $\lambda_1 = 0$ ($\lambda_1 = 1$). In other words, either both cities offer the same real wage or, concentration in the city that offers the higher real wage emerges as an equilibrium.

Proposition 1. *Concentration of population in one city is an equilibrium if and only if congestion costs are sufficiently low with respect to transport costs and to the level of human capital there, namely, if and only if*

$$\gamma \leq \tau(\frac{2\sigma - 1}{\sigma}) + ln(1 + k_1^h), \tag{15}$$

where k_1^h is the human capital achieved through education in city 1.

Proof:

By substituting $\lambda_1 = 1$ and $\lambda_2 = 0$ in equations (10) to (14) and calculating the real wages, we have that $\omega_1 \geq \omega_2$ if and only if the above expression holds. □

Hence, we have that congestion cost is a centrifugal force that works against concentration since, the higher the value of γ, the less likely is concentration of population in one city. In other words, the higher the congestion costs in a large city, the more interested its citizens will be in moving to a smaller city nearby where congestion is lower. In this way, the model we have proposed makes more sense in the context of metropolitan areas. On the other hand, the higher the transport cost between the cities, the more interested individuals are in staying in the large city because there they can obtain more goods. It

is important to remark that it is not the absolute value of the parameters of congestion and transportation that produces concentration or dispersion, but the relative effect the one over the other.

What differentiates this result from previous works[11] is that human capital is an element which attracts people to the largest cities, that is where more high–skilled workers can be found. In the metropolis, we cannot only find a large number of specialized shopping centers, but also the place where we can find more services of high–skilled workers, such as attorneys in law services, medical centers or theaters, all of which make the larger city attractive.

Unfortunately, the model is too complicated to solve analytically in other cases. Hence, at this point we are driven to numerical examples. In the following figures, we will represent the population size of city 1 in the x axis and the differential curve of real wages between cities 1 and 2 , in the y axis. By considering now that the high–skilled workers initially constitute 50% of population, we can obtain the following situation:

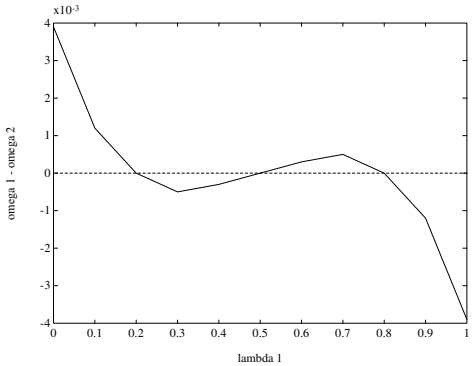


Fig. 4

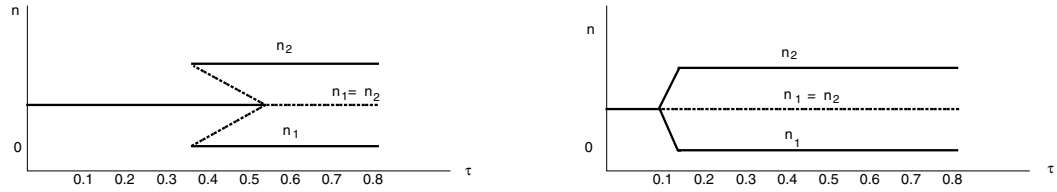
We can see that two cities of different sizes can coexist as a stable configuration,[12] one with 20% of total population and the other 80%. Therefore, the introduction of human

[11] See Krugman (1992), for example, or Alonso–Villar (1996), which is basically the same work but without human capital.

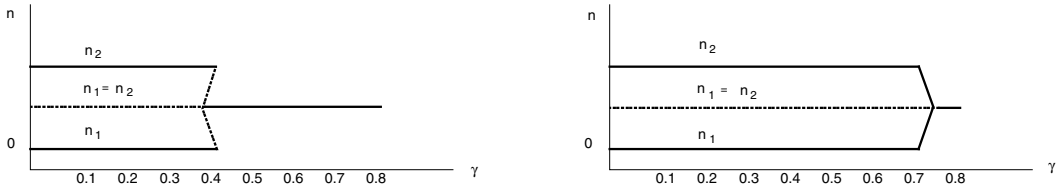
[12] All simulations assume the following values for the parameters: $\sigma = 4, c_1 = 0.4, c_2 = 0.1$ and $\pi = 2$.

capital leads to new asymmetric ($\lambda_1 \neq \frac{1}{2}$) stable equilibria.[13] This makes the model more realistic and richer. In particular, the introduction of human capital allows the existence of a large city together with a small one, a typical configuration of metropolitan areas.

Finally, in Figs. 5 and 6 we show, for different levels of human capital (25% for case a and 75% for case b), what happens with these asymmetric configurations when τ and γ change. We plot the number of firms in each location as functions of τ and γ, respectively. The central line ($n_1 = n_2$) represents the dispersed equilibrium, which is stable where solid, and unstable where dashed. The upper and lower lines give the equilibria where uneven distributions emerge. Total concentration is always stable, while the other asymmetric distributions can be stable or unstable.



Figs. 5a and 5b. $\gamma = 0.8$



Figs. 6a and 6b. $\tau = 0.1$

These figures confirm what we obtained before: transport costs and human capital foster agglomeration while congestion costs works against it. On the other hand, the

[13] This kind of configuration is also obtained by Brakman et al. (1996) by considering the negative effects derived from industrial concentration.

analysis suggests that an increase in the human capital level enlarges the region of values of τ and γ within which cities of different size can coexist as a stable configuration. Furthermore, the higher the human capital level, the lower (higher) the value of τ (γ) under which these asymmetric equilibria emerge. We can explain this as follows. In spite of the small savings on transport costs that concentration involves when τ is small, the agglomeration of human capital can be a stronger centripetal force and drive the economy to concentration if congestion costs are not too high. If the advantages of agglomeration are important (i.e., if human capital is important) and congestion costs take an intermediate value, a large city along with a small one nearby is the most similar pattern to concentration in only one city. On the contrary, when congestion (transport) costs are high enough, the above configuration cannot be sustained anymore and even distribution (concentration) emerges as the only possible configuration pattern. Then, if congestion costs take an intermediate value, and transport costs are low, human capital is the factor that most influences the final configuration, i.e., whether the case of two cities of different sizes can be sustained or not.

4. The role of the government

In this section we will determine whether the amount of education reached in equilibrium is efficient. As we did in the calculation of the short–run equilibrium, we must consider three cases. First, we will write the expression which gives us the number of firms that enter the market in city j when each high–skilled worker educates m_j low–skilled workers. The amount of a good that any firm produces does not depend on the level of human capital (see Eq. (6)), but the goods' variety, n_j, does. Therefore, the efficient level of education will be the one that maximizes the value of n_j

$$n_j = \frac{\lambda_j^h(1-c_2 m_j) + (\lambda_j^l - \lambda_j^h m_j)\frac{1}{\pi} + \lambda_j^h m_j(1-c_1)}{\alpha\sigma(1+\lambda_j^h+\lambda_j^h m_j)^{-1}}. \tag{16}$$

The derivative of this expression with respect to m_j is:

$$\frac{\partial n_j}{\partial m_j} = \frac{\lambda_j^h}{\alpha()}[1-c_1-\frac{1}{\pi}-c_2-\alpha\prime()n_j\sigma]. \tag{17}$$

The following cases have to be considered:

- $P_j^{e*} > P_j^{e**} \iff c_1 + c_2 < 1 - \frac{1}{\pi}$. In this case, the above derivative is positive and, therefore, the level of education should be the highest possible. Then, the education level in equilibrium is efficient.

- $P_j^{e*} < P_j^{e**} \iff c_1 + c_2 > 1 - \frac{1}{\pi}$. By evaluating the above derivative at the equilibrium ($m_j = 0$) we can show that if the value of $c_1 + c_2$ is close enough to $1 - \frac{1}{\pi}$, then the derivative is positive. This means that by increasing the level of education reached in equilibrium, more goods would be produced in the city. Therefore, the equilibrium is not efficient.

- $P_j^{e*} = P_j^{e**} \iff c_1 + c_2 = 1 - \frac{1}{\pi}$. In this latter case, the above derivative is positive, which implies that the level of education should be the highest possible. We can thus see that optimum and equilibrium can differ (in this case we have multiplicity of equilibria, as we mentioned in section 2).

We have shown that the level of education which emerges as equilibrium can be lower than the efficient level when the cost of time in which teachers and pupils incur is sufficiently high. In other words, the number of goods the economy offers can increase if the level of education rises. Since human capital brings a positive externality on the economy, government can be introduced in the model in order to internalize it. The important point to note is that a change in the education level of the economy can affect the spatial distribution of population (and production). Therefore, by raising the human capital level of the economy the government not only internalizes this externality but may also affect the spatial configuration.

In this section, we will assume that the government is able to encourage individuals to educate or be educated by deciding the amount of individuals,[14] m, that every high–skilled worker should educate in order to correct this externality.[15]

[14] Think, for example, of the possibility of training courses for old people.

[15] Our aim is not to analyze the way these investments can be undertaken. In this vein, Zhang (1996) presents an endogenous growth model with externalities where optimal public investments in human

Later on, we will see whether its effect over the spatial configuration of the economy is positive or negative. The optimal amount of education only depends on the costs of education (c_1, c_2) due to the time that both types of individuals lose in the education process, as well as on the productivity advantages that the high–skilled workers have associated as compared with the low–skilled workers (π). Because these parameters are common to both cities, and the initial proportion of human capital in each city is the same, the optimal proportion of education (the number of pupils per teacher) will be equal in the two cities. This makes the model easier to work with.

Despite this common optimal value for education in the two cities, it is worth noting that investments in education can drive the economy to different long–run equilibria and, therefore, to different welfare levels, depending on the moment in which the government carries out the policy. For example, if initially 25% of population is composed of high–skilled workers,[16] $\tau = 0.5$ and $\gamma = 1$ we have the following curves of real wages depending on the level of education the government chooses.

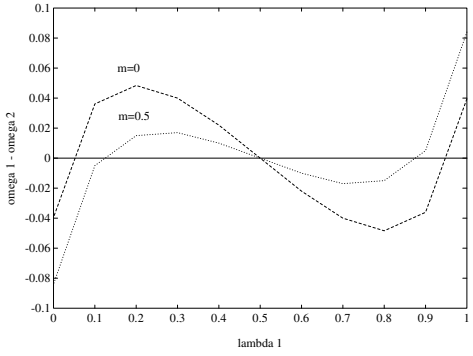


Fig. 7

If the initial distribution of population between the two locations were such that λ_1 took a value between the two unstable equilibria in the left side of Fig. 7 (between 0.06 and 0.15, approximately), we could reach the following two situations. Let us consider first the case where the government immediately carries out an improvement in education

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ca$_{\text{p}}$ital throu$_{\text{g}}$h subsidizin$_{\text{g}}$ p$_{\text{r}}$ivate education or p$_{\text{r}}$ovidin$_{\text{g}}$ p$_{\text{u}}$blic education are considered.

[16] For the values of the costs of education that we assume o$_{\text{p}}$timum and equilibrium do not coincide.

($m = 0.5$). In this case, concentration in city 2 emerges as the long–run equilibria. Secondly, let us suppose that the economy evolves in such a way that population in city 1 increases enough (λ_1 higher than 0.2, for example) before the government changes the level of education. In this situation, the same increase in education does not affect the final distribution of people across cities and an even distribution emerges again as the long–run equilibrium. Therefore, if one of the two cities is very small a change in education can dramatically affect its future. If the government wants to favor dispersion, it has to wait (before raising the education level) until the smallest city becomes large enough.

Thus, we have shown that when the model has multiplicity of equilibria, for example two long–run equilibria, the government may alter the spatial configuration by encouraging education too soon.

Due to the difficulty in finding analytical solutions for the system defined by equations (10) to (14), it is not easy to compare, from the social welfare point of view, the long–run equilibria that appear before and after an improvement in education in a general way. However, we can do so in some simpler cases, as we can see in the next proposition.

Proposition 2. *Assume that* $c_2 + c_1 = 1 - \frac{1}{\pi}$ *and, therefore, that the level of education in equilibrium is inefficient.*

If concentration is initially a stable equilibrium then, after the optimal investment in education undertaken by the government to correct the externality, concentration will continue being a stable equilibrium and individual's utility will have improved.

Moreover, if even distribution is initially a stable equilibrium and after the education improvement the emergent equilibrium is either an even distribution or concentration, individual's utility will have improved.

Proof:

By using proposition 1, we already know that if concentration is initially an equilibrium, after an improvement in education, the level of human capital will increase and, therefore, concentration will be more likely. Hence, if the efficient investment is such

that the high–skilled workers should educate the maximum feasible number of low–skilled workers, concentration will still be in equilibrium.

On the other hand, when individuals are either agglomerated in one city, or evenly distributed between the two cities, the wage rates of high skilled workers are equal to 1. This implies that, under the assumption of $c_1 + c_2 = 1 - \frac{1}{\pi}$, the nominal incomes of both high and low–skill workers have the same value before and after the change in education. Hence, after an improvement in education all individuals will keep their nominal income. Since the number of goods that this economy can produce increases with education, it follows that the price that every firm charges must decrease in order to sell the whole production (which does not depend on the human capital level). This implies that the utility level of all individuals of this economy increases (the number of goods increases, prices decrease, and workers' nominal incomes do not change) by investing in education. □

5. Conclusions

In this paper we have developed a general equilibrium model that attempts to explain the formation of metropolitan areas in a context of monopolistic competition. We have shown that increasing returns to scale, the existence of intercities transportation costs, and the level of human capital are factors which favor the concentration of the economic activity. Besides, as opposed to the standard urban models, the external effects of human capital appear as the motor of growth in cities, as the recent theories of economic growth suggest. On the other hand, the existence of congestion costs puts a stop to such agglomerations.

A result that can be inferred from the model is that, in a national context, the higher the human capital in a region, the higher the number of firms can be supported there, but also the higher its ability to attract firms from other regions.

In contrast with the model without human capital, we have shown not only that an even distribution of population between the two location is, in addition to concentration, a possible stable equilibrium, but also that there are other asymmetric stable equilibria.

This makes the model richer and more realistic. Hence, two cities of different size can coexist, as, for example, a large city together with a small one, which is the typical configuration pattern in the context of of metropolitan areas. We have also obtained that this kind of equilibria is more likely to appear when the level of human capital is high and when congestion costs take intermediate values and transport costs low values. In this way, total concentration is impossible because of congestion costs, but simultaneously, an even distribution of the population is not necessary.

Since education involves a positive externality in the model, we have introduced the government as one more economic agent with the ability of encouraging both high–skilled individuals to educate low–skilled workers, and low–skilled workers to acquire education, when the market does not achieve the efficient result by itself. There is scope for the government for two reasons. Firstly, to correct this externality and secondly, to affect the spatial configuration. We have underlined that an investment in education can lead the economy to different long–run equilibria, and therefore, to different welfare levels, depending on the timing of these investments in education. An increase in the human capital level can have negative effects on the spatial distribution of population because it fosters agglomeration. However, we have also shown some cases where an improvement in education drives the economy to a better spatial configuration from the social–welfare point of view.

Appendix

In order to obtain the wage rate of high–skilled workers, we will begin by solving the following problem:

$$\begin{aligned} &\max \left(\sum_i {c_i^k}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \\ &\text{s.t. } \sum_i p'_{ik} c_i^k = y, \end{aligned} \tag{18}$$

where c_i^k is the consumption of good i by an individual of city k, p'_{ik} is the c.i.f. price paid by this individual living in city k for a unit of good i, and y is this individual's income.

By calculating the first order conditions, we have

$$c_i^k = \frac{{p'_{2k}}^{\sigma}}{{p'_{ik}}^{\sigma}} c_2^k. \tag{19}$$

This equation can be rewritten as follows

$$p'_{ik} c_i^k = \frac{{p'_{2k}}^{\sigma}}{{p'_{ik}}^{\sigma-1}} c_2^k. \tag{20}$$

Taking into account that, first, the consumption of good i in city k is the sum of the different consumptions of its citizens, which we denote by C_i^k, and that, second, the previous equation holds for any individual of city k, we have

$$p'_{ik} C_i^k = \frac{{p'_{2k}}^{\sigma}}{{p'_{ik}}^{\sigma-1}} C_2^k. \tag{21}$$

We define Y_k as the income of city k. Since the price of education paid by low–skilled workers (pupils) goes to high–skilled workers (educators) we can write this income as

$$Y_k = \lambda_k^h (1 - c_2 m_k) W_k + (\lambda_j^l - m_k \lambda_k^h) \frac{W_k}{a} + m_k \lambda_k^h (1 - c_1) W_k, \tag{22}$$

where m_k is the number of pupils that every teacher of city k educates.

On the other hand, this income is used to pay for goods consumed in city k, namely, $Y_k = \sum_i p'_{ik} C_i^k$. Combining this with expression (21) yields

$$Y_k = p'_{2k} C_2^k \left[\sum_i \left(\frac{p'_{2k}}{p'_{ik}} \right)^{\sigma-1} \right]. \tag{23}$$

Then,

$$p'_{2k}C_2^k = \frac{Y_k {p'_{2k}}^{1-\sigma}}{\sum_j {p'_{jk}}^{1-\sigma} n_j}. \tag{24}$$

Let S_{2k} be expenditures in city k on goods produced in city 2, namely, $S_{2k} = n_2 p'_{2k} C_2^k$ (we are identifying good 2 with any good produced in city 2). If we introduce expression (24) in S_{2k}, then we add in k and use that $p'_{jk} = p_{jk}\, e^{(\tau D_{jk} + \gamma \lambda_k)}$, we have that the expenditures of city 2 take the form

$$\sum_k S_{2k} = \frac{\sum_k n_2 Y_k \left[w_2 (1 + \lambda_2^h + \lambda_2^h m_2)^{-1} e^{\tau D_{2k} + \gamma \lambda_k} \right]^{1-\sigma}}{\sum_j n_j [w_j (1 + \lambda_j^h + \lambda_j^h m_j)^{-1} e^{\tau D_{jk} + \gamma \lambda_k}]^{1-\sigma}}. \tag{25}$$

On the other hand, revenues in each city have to equate income which means that

$$\sum_k S_{2k} = Y_2. \tag{26}$$

Using equations (22), (25) and (26), and taking into account that the number of firms which enter the market in city j is

$$n_j = \frac{\lambda_j^h (1 - c_2 m_j) + (\lambda_j^l - \lambda_j^h m_j)\frac{1}{\pi} + \lambda_j^h m_j (1 - c_1)}{\alpha\, \sigma (1 + \lambda_j^h + \lambda_j^h m_j)^{-1}}, \tag{27}$$

we find that the wage rate in city 2 is

$$w_2 = \left\{ (1 + k_2^h)^\sigma \sum_k Y_k [e^{-(\tau D_{2k} + \gamma \lambda_k)} T_k]^{\sigma - 1} \right\}^{\frac{1}{\sigma}}, \tag{28}$$

where T_k, the price index in city k, can be written as follows

$$T_k = \left[\sum_j \overline{\lambda}_j (1 + k_j^h)^\sigma (w_j\, e^{\tau D_{jk} + \gamma \lambda_k})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{29}$$

and $\overline{\lambda}_j = \lambda_j^h (1 - c_2 m_j) + (\lambda_j^l - \lambda_j^h m_j)\frac{1}{\pi} + \lambda_j^h m_j (1 - c_1)$ being the units of high–skilled labor available for production. Analogous expressions hold for a generic city j.

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